Prelim Examination 2015/2016
(Assessing all 3 Units)

MATHEMATICS

CFE Advanced Higher Grade

Time allowed - 3 hours

Total marks – 100

Attempt ALL questions.

You may use a calculator.
Full credit will be given only to solutions which contain appropriate working.
State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.
Use **blue** or **black** ink.
Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\ln x,\ x>0$	$\dfrac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)dx$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)+c$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+c$
$\sec^2(ax)$	$\dfrac{1}{a}\tan(ax)+c$
e^{ax}	$\dfrac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $S_n = \frac{1}{2}n[2a+(n-1)d]$

(Geometric series) $S_n = \dfrac{a(1-r^n)}{1-r}$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

De Moivre's theorem

$$[r(\cos\theta+i\sin\theta)]^n = r^n(\cos n\theta+i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |a||b|\sin\theta\hat{n} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Answer all the questions.

1. Find the term in a^{-3} in the expansion of $\left(4a^2 + \frac{3}{a}\right)^6$. **4**

2. Given the equation z + 2i$\bar{z}$ = 8 + 7i, express z in the form a + ib. **4**

3. (a) Differentiate and simplify $4\tan^{-1}\sqrt{1-x}$, where $x < 1$. **3**

(b) Use the substitution $u = \cos\theta - 1$ to evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin\theta}{(\cos\theta - 1)^4}\, d\theta$ **5**

4. (a) Prove that $\sqrt{2}$ is irrational. **4**

(b) Consider the following two statements S and T:

S: If p and q are two odd prime numbers then p + q is not prime.

T: If p and q are two odd prime numbers then p – q is not prime.

For each of S and T, give a proof if it is true, or give a counter example if it is false. **3**

5. Given that $-2 + 5i$ is a root of the equation $z^3 + 6z^2 + 37z + 58 = 0$, find the other roots. **3**

6. The radius of a sphere is increasing at a rate of 3 cm/s.

Find, in terms of π, the rate at which the volume of the sphere is increasing when the radius is 5cm.

[You may assume that the volume of a sphere is given by: $V = \frac{4}{3}\pi r^3$.] **4**

7. (a) Write down the 2×2 matrix R representing reflection in the line $y=-x$. **1**

(b) Write down the 2×2 matrix S representing an anticlockwise rotation of 90^0 about the origin. **1**

8. A curve is defined by the parametric equations

- $x=8t$
- $y=t^3-27t+50$ for all t.

Find the coordinates of the stationary points of this curve and, by considering $\frac{d^2y}{dx^2}$, determine their nature. **7**

9. Let:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}$$

Show that AB = k***I*** for some constant k, where ***I*** is the 3x3 identity matrix.

Hence obtain:

i) The inverse matrix A^{-1}

ii) The matrix A^2B 4

10. A function is defined implicitly by $e^{2x+3y}=x^2-\ln\left(xy^3\right)$.

Find, in terms of x and y, a formula for $\frac{dy}{dx}$. **4**

11. Solve the differential equation:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x$$

given that y = 2 and $\frac{dy}{dx}$ = 1, when x = 0. **10**

12. The function f is defined by $f(x) = ax^3 + bx^2 + cx + 6$ where a, b and c are constants.

It is known that the graph of f passes through the point $(1, 7)$ and has a stationary point at $(-1, 7)$.

(a) Deduce that a, b and c must satisfy the system of equations

$$\begin{aligned} a + \ \ b + c &= 1 \\ 3a - 2b + c &= 0 \\ a - \ \ b + c &= -1. \end{aligned}$$

4

(b) Use Gaussian elimination to find the values of a, b and c. **5**

13. (a) Show that

$$\frac{1}{\text{cosec}x\text{cosec}2x} = 2\cos x \sin^2 x.$$

1

(b) Use integration by parts to show that

$$\int \cos x \sin^2 x dx = \frac{1}{3}\sin^3 x + c.$$

2

(c) Hence, or otherwise, find the particular solution of the differential equation

$$\frac{dy}{dx} = e^x \text{cosec}2y\text{cosec}y,$$

given that $y = \frac{\pi}{6}$ when $x = 0$. **4**

14. Evaluate $\displaystyle\int_1^3 \frac{2x^3 - 3x^2 - 3}{x^2(x^2 + 1)} dx.$ **8**

15. Water is being heated in a kettle.
At time t seconds, the temperature of the water is $T\ {}^{0}$C.

The rate of increase of the temperature of the water at any time t is given by the differential equation

$$\frac{dT}{dt} = k(130 - T), \quad T \leq 100$$

where k is a positive constant.

(a) Given that $T = 25$ when $t = 0$, show that

$$T = -105e^{-kt} + 130.$$ **4**

(b) When the temperature of the water reaches 100 ^{0}C, the kettle switches off.

Given that $k = 0.009$, find the time, to the nearest second, when the kettle switches off. **2**

16. Let $z = \cos\theta + i\sin\theta.$

(a) Use de Moivre's theorem to express z^5 in terms of $5\theta.$ **1**

(b) Use the binomial theorem to express z^5 in terms of $\sin\theta$ and $\cos\theta.$ **2**

(c) Hence show that

(i) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ **2**

(ii) $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$ **2**

(d) Use your answers to (c)(i) and (c)(ii) to show that

$$\cot 5\theta = \frac{1 - 10\tan^2\theta + 5\tan^4\theta}{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}.$$ **4**

[*END OF QUESTION PAPER*]

Marking Scheme - CFE Advanced Higher Grade 2015/2016
Prelim (Assessing all 3 Units)

<table>
<tr><th></th><th>Give one mark for each •</th><th>Illustrations for awarding each mark</th></tr>
<tr><td>1</td><td>ans: $5832a^{-3}$ 4 marks

• finds correct general term
• simplifies to find correct expression for power of a
• solves for r correctly
• finds correct term</td><td>• $\binom{6}{r}\left(4a^2\right)^{6-r}\left(\frac{3}{a}\right)^r$
• a^{12-3r}
• $r=5$
• $5832a^{-3}$</td></tr>
<tr><td>2</td><td colspan="2">
<table>
<tr><th>Part</th><th>Marks</th><th>Level</th><th>Calc.</th><th>Content</th><th>Answer</th><th>U2 OC3</th></tr>
<tr><td></td><td>4</td><td>C</td><td>CN</td><td>A16, A17</td><td></td><td>2005 Q9</td></tr>
</table>
Let $z = a + ib$ so $\bar{z} = a - ib$ 1
$z + 2i\bar{z} = 8 + 7i$
$a + ib + 2ia + 2b = 8 + 7i$ 1
$a + 2b = 8$
$2a + b = 7$ M1
$3a = 6$
$a = 2;\ b = 3$
$z = 2 + 3i.$ 1
</td></tr>
<tr><td>3(a)</td><td>ans: $\frac{-2}{(2-x)\sqrt{1-x}}$ 3 marks

• differentiates $\tan^{-1}$ correctly

• continues correctly

• simplifies correctly</td><td>• $\ldots\frac{1}{1+\sqrt{1-x}^{\,2}}\ldots$
• $4\ldots\frac{1}{2}(1-x)^{-\frac{1}{2}}(-1)$
• $\frac{-2}{(2-x)\sqrt{1-x}}$</td></tr>
<tr><td>3(b)</td><td>ans: $\frac{7}{3}$ 5 marks

• starts correctly
• substitutes correctly

• integrates correctly</td><td>• $du = -\sin\theta d\theta$
• $-\int\frac{du}{u^4}$
• $\left[\frac{1}{3u^3}\right]_{\ldots}^{\ldots}$</td></tr>
</table>

<table>
<tr>
<td></td>
<td>

- substitutes correctly

- correct answer

</td>
<td>

- $\left(\frac{1}{3(-1)^3}\right)-\left(\frac{1}{3\left(\frac{-1}{2}\right)^3}\right)$
- $\frac{7}{3}$

</td>
</tr>
<tr>
<td>4(a)</td>
<td colspan="2">
<table>
<tr><th>Part</th><th>Marks</th><th>Level</th><th>Calc.</th><th>Content</th><th>Answer</th><th>U3 OC2</th></tr>
<tr><td>(a)</td><td>4</td><td>C</td><td></td><td>P2</td><td></td><td>1996 SY2 Q10</td></tr>
</table>

(a) Suppose that $\sqrt{2} = \frac{m}{n}$ where the integers m, n have no common factor. **1**
Then $m^2 = 2n^2$. **1**
Thus m is even, so $m = 2u$ for some integer u. **1**
Thus $4u^2 = 2n^2$ i.e. $n^2 = 2u^2$, so n is also even, **contradicting** the assumption. **1**
</td>
</tr>
<tr>
<td>4(b)</td>
<td colspan="2">
<table>
<tr><th>Part</th><th>Marks</th><th>Level</th><th>Calc.</th><th>Content</th><th>Answer</th><th>U3 OC5</th></tr>
<tr><td></td><td>3</td><td>C</td><td>CN</td><td>P6, P1</td><td></td><td>2000 SY2 Q5</td></tr>
</table>

S is true. If p and q are two odd primes then $p + q$ is even. **1**
Since odd primes are greater than or equal to 3, $p + q$ cannot be 2. **1**

T is false. For example $p = 5, q = 3$. **1**
(Other examples will do, but they *must* differ by 2.)
</td>
</tr>
<tr>
<td>5</td>
<td>

ans: $-2-5i; -2$ **3 marks**

- starts correctly
- correct root
- correct root

</td>
<td>

- $-2+5i/1 \quad 6 \quad 37 \quad 58$
- $-2-5i$
- -2

</td>
</tr>
</table>

	Give one mark for each •	Illustrations for awarding each mark
6	**ans:** 300π cm^3/s **4 marks** • starts correctly • continues correctly • continues correctly • correct answer	• $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ • $4\pi r^2\times...$ • $12\pi r^2$ • 300π
7(a)	**ans:** $R=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ **1 mark** • correct answer	• $\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$
7(b)	**ans:** $S=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ **1 mark** • correct answer	• $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

	Give one mark for each •	Illustrations for awarding each mark
8	**ans:** $(-24,104)\rightarrow Maximum T.P.$ & $(24,-4)\rightarrow Minimum T.P.$ **7 marks** • find correct first derivative • solves for t correctly • correct coordinates • correct coordinates • finds second derivative correctly • correct nature • correct nature	• $\frac{dy}{dx}=\frac{3t^2-27}{8}$ • $\frac{dy}{dx}=0\Rightarrow t=\pm3$ • $(-24,104)$ • $(24,-4)$ • $\frac{d^2y}{dx^2}=\frac{3t}{32}$ (or equivalent) • $(-24,104)\rightarrow\frac{d^2y}{dx^2}<0$ $Maximum T.P.$ • $(24,-4)\rightarrow\frac{d^2y}{dx^2}>0$ $Minimum T.P.$

	Give one mark for each •	Illustrations for awarding each mark
9		Part	Marks	Level	Calc.	Content	Answer	U3 OC2
4	C	CN	A25	$k=2$, $A^{-1}=\frac{1}{2}B$, $A^2B=2A$	2001 B3	 *[No marking instructions available]*	
10	**ans:** $\frac{dy}{dx}=\frac{2x-\frac{1}{x}-2e^{2x+3y}}{3e^{2x+3y}+\frac{3}{y}}$ **4 marks**	
	• starts correctly	• $e^{2x+3y}\left(\frac{d}{dx}(2x+3y)\right)=\ldots$
	• continues correctly	• $\ldots=2x-\frac{1}{xy^3}\left(\frac{d}{dx}(xy^3)\right)$
	• differentiates correctly	• $2e^{2x+3y}+3e^{2x+3y}\frac{dy}{dx}=2x-\frac{y^3+3xy^2\frac{dy}{dx}}{xy^3}$
	• correct answer	• $\frac{dy}{dx}=\frac{2x-\frac{1}{x}-2e^{2x+3y}}{3e^{2x+3y}+\frac{3}{y}}$ (or equivalent)

<table>
<tr><th></th><th>Give one mark for each •</th><th>Illustrations for awarding each mark</th></tr>
<tr><td>11</td><td colspan="2">
<table>
<tr><th>Part</th><th>Marks</th><th>Level</th><th>Calc.</th><th>Content</th><th>Answer</th><th>U3 OC4</th></tr>
<tr><td></td><td>10</td><td>C</td><td>CN</td><td>DE7, DE3</td><td>$(1-2x)e^{2x}+e^{x}$</td><td>2003 B6</td></tr>
</table>
<table>
<tr><td>
•1 form auxiliary equation
•2 solve auxiliary equation
•3 state complementary function
•4 interpret form of particular integral
•5 state first and second derivatives
•6 solve for coefficients
•7 state general solution
•8 differentiate general solution
•9 interpret initial condition
•10 solve for coefficients
</td><td>
•1 $\lambda^2-4\lambda+4=0$
•2 $\lambda=2$
•3 $y_c=(A+Bx)e^{2x}$
•4 $y_p=ae^{x}$
•5 $y'_p=y''_p=ae^{x}$
•6 $a=1$
•7 $y=(A+Bx)e^{2x}+e^{x}$
•8 $\frac{dy}{dx}=Be^{2x}+2(A+Bx)e^{2x}+e^{x}$
•9 $2=A+1, 1=B+2A+1$
•10 $A=1, B=-2$
</td></tr>
</table>
</td></tr>
<tr><td>12(a)</td><td>ans: Proof 4 marks

• obtains first equation
• differentiates correctly
• obtains second equation
• obtains third equation</td><td>

• $a+b+c=1$
• $f'(x)=3ax^2+2bx+c$
• $3a-2b+c=0$
• $a-b+c=-1$</td></tr>
<tr><td>12(b)</td><td>ans: $a=1, b=1, c=-1$ 5 marks

• correct augmented matrix

• correct first modified system

• correct second modified system

• correct third modified system

• correct solution</td><td>
• $\begin{pmatrix}1 & 1 & 1 & 1\\ 3 & -2 & 1 & 0\\ 1 & -1 & 1 & -1\end{pmatrix}$
• $\begin{pmatrix}1 & 1 & 1 & -1\\ 0 & -5 & -2 & -3\\ 1 & -1 & 1 & -1\end{pmatrix}$
• $\begin{pmatrix}1 & 1 & 1 & -1\\ 0 & -5 & -2 & -3\\ 0 & -2 & 0 & -2\end{pmatrix}$
• $\begin{pmatrix}1 & 1 & 1 & -1\\ 0 & -5 & -2 & -3\\ 0 & 0 & 4 & -4\end{pmatrix}$
• $a=1, b=1, c=-1$</td></tr>
</table>

13(a)	**ans:** Proof **1 mark** • proves correctly	• $\frac{1}{\text{cosec}x\text{cosec}2x} = \sin x \sin 2x =$ $\sin x(2\sin x\cos x) = 2\cos x\sin^2 x.$
13(b)	**ans:** Proof **2 marks** • starts correctly • completes proof	• $\int \cos x\sin^2 xdx =$ $\sin x.\sin^2 x - \int \sin x(2\sin x\cos x)dx$ $\ldots = \sin^3 x - \int 2\sin^2 x\cos xdx \Rightarrow$ • $3\int \cos x\sin^2 xdx = \sin^3 x \Rightarrow$ $\int \cos x\sin^3 xdx = \frac{1}{3}\sin^3 x + c$
13(c)	**ans:** $y = \sin^{-1}\left(\sqrt[3]{\frac{3}{2}e^x - \frac{11}{8}}\right)$ **4 marks** • rearranges & starts to integrate • substitutes & integrates correctly • substitutes to find correct constant • correct particular solution	• $\int \frac{dy}{\cos ec2y\cos ecy} = \int e^x dx$ $2\int \cos y\sin^2 ydy =$ • $\int e^x dx \Rightarrow 2\left(\frac{1}{3}\sin^3 y\right) = e^x + c$ • $\left(0, \frac{\pi}{6}\right): \left(\sin\frac{\pi}{6}\right)^3 = \frac{3}{2}e^0 + c \Rightarrow c = -\frac{11}{8}$ • $y = \sin^{-1}\left(\sqrt[3]{\frac{3}{2}e^x - \frac{11}{8}}\right)$
14	**ans:** $-2 + \ln 5$ **8 marks** • starts correctly • continues correctly • a correct constant • all constants correct • substitutes correctly • integrates correctly • substitutes correctly • correct answer	• $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$ • $2x^3 - 3x^2 - 3 \equiv$ $Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2$ • $A = 0, B = -3, C = 2 or D = 0$ • $A = 0, B = -3, C = 2 \,\&\, D = 0$ • $\int_1^3 \left(\frac{-3}{x^2} + \frac{2x}{x^2 + 1}\right)dx$ • $\left[\frac{3}{x} + \ln\|x^2 + 1\|\right]$ • $(1 + \ln 10) - (3 + \ln 2)$ • $-2 + \ln 5$

	Give one mark for each •	Illustrations for awarding each mark
15(a)	**ans:** Proof **4 marks** • separate the variables & know to integrate • integrate correctly • substitute correctly to find correct constant • completes proof	• $\int \frac{dt}{130-t} = \int k\,dt$ • $-\ln\lvert 130-T\rvert = kt+c$ • $T = 130 - \frac{1}{Ae^{kt}} \Rightarrow 25 = 130 - \frac{1}{Ae^{0}}$ $\Rightarrow A = \frac{1}{105}$ (or equivalent) • $T = 130 - \frac{1}{\frac{1}{105}e^{kt}} \Rightarrow \ldots \Rightarrow T = -105e^{-kt} + 130$
15(b)	**ans:** 139 seconds **2 marks** • substitutes correctly • correct answer	• $100 = -105e^{-0.009t} + 130$ • 139 seconds

	Give one mark for each •	Illustrations for awarding each mark
16(a)	**ans:** $z^5 = \cos 5\theta + i\sin 5\theta$ **1 mark** • correct answer	• $\cos 5\theta + i\sin 5\theta$
16(b)	**ans:** $z^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta$ $-10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$ **2 marks** • expands correctly • simplifies correctly	• $(\cos\theta)^5 + 5(\cos\theta)^4(i\sin\theta) +$ $10(\cos\theta)^3(i\sin\theta)^2 + 10(\cos\theta)^2(i\sin\theta)^3 +$ $5(\cos\theta)(i\sin\theta)^4 + (i\sin\theta)^5$ • $\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta$ $-10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$
16(c)(i)	**ans:** Proof **2 marks** • equates real parts correctly • correct expression	• $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ • $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
16(c)(ii)	**ans:** Proof **2 marks** • equates imaginary parts correctly • correct expression	• $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ • $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$
16(d)	**ans:** Proof **4 marks** • starts correctly • simplifies correctly • substitutes correctly • completes proof	• $\dfrac{\cos 5\theta}{\sin 5\theta}$ • $\dfrac{16 - 20\sec^2\theta + 5\sec^4\theta}{16\tan^5\theta - 20\tan^3\theta\sec^2\theta + 5\tan\theta\sec^4\theta}$ • $\dfrac{16 - 20(1+\tan^2\theta) + 5(1+\tan^2\theta)^2}{\left[\begin{array}{l}16\tan^5\theta - 20\tan^3\theta(1+\tan^2\theta) + \\ 5\tan\theta(1+\tan^2\theta)^2\end{array}\right]}$ • $\ldots = \dfrac{1 - 10\tan^2\theta + 5\tan^4\theta}{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}$

TOTAL MARKS = 100

Additional Questions Solutions form Quest

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	CN	A16, A17		2005 Q9

Let $z = a + ib$ so $\bar{z} = a - ib$ **1**

$$z + 2i\bar{z} = 8 + 7i$$

$$a + ib + 2ia + 2b = 8 + 7i \quad \textbf{1}$$

$$a + 2b = 8$$

$$2a + b = 7 \quad \textbf{M1}$$

$$3a = 6$$

$$a = 2;\ b = 3$$

$$z = 2 + 3i. \quad \textbf{1}$$

Part	Marks	Level	Calc.	Content	Answer	U3 OC5
	3	C	CN	P6, P1		2000 SY2 Q5

S is true. If p and q are two odd primes then $p + q$ is even. **1**
Since odd primes are greater than or equal to 3, $p + q$ cannot be 2. **1**

T is false. For example $p = 5, q = 3$. **1**
(Other examples will do, but they *must* differ by 2.)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	4	C		P2		1996 SY2 Q10

(a) Suppose that $\sqrt{2} = \frac{m}{n}$ where the integers m, n have no common factor. **1**
Then $m^2 = 2n^2$. **1**
Thus m is even, so $m = 2u$ for some integer u. **1**
Thus $4u^2 = 2n^2$ i.e. $n^2 = 2u^2$, so n is also even, contradicting the assumption. **1**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	CN	A25	$k = 2, A^{-1} = \frac{1}{2}B, A^2B = 2A$	2001 B3

[No marking instructions available]

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	10	C	CN	DE7, DE3	$(1-2x)e^{2x}+e^{x}$	2003 B6

•[1] form auxiliary equation	•[1] $\lambda^2 - 4\lambda + 4 = 0$
•[2] solve auxiliary equation	•[2] $\lambda = 2$
•[3] state complementary function	•[3] $y_c = (A + Bx)e^{2x}$
•[4] interpret form of particular integral	•[4] $y_p = ae^{x}$
•[5] state first and second derivatives	•[5] $y'_p = y''_p = ae^{x}$
•[6] solve for coefficients	•[6] $a = 1$
•[7] state general solution	•[7] $y = (A + Bx)e^{2x} + e^{x}$
•[8] differentiate general solution	•[8] $\frac{dy}{dx} = Be^{2x} + 2(A + Bx)e^{2x} + e^{x}$
•[9] interpret initial condition	•[9] $2 = A + 1, 1 = B + 2A + 1$
•[10] solve for coefficients	•[10] $A = 1, B = -2$