Factorials and the Binomial Theorem

Skill	Achieved ?
Know that the ***factorial of n*** ($n \in \mathbb{N}$) (aka ***n factorial*** or ***factorial n***) is: $$n! \overset{def}{=} n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$	
Know that $1! = 1$ and, by convention: $$0! \overset{def}{=} 1$$	
Calculate factorials such as $4!$ and $11!$	
Know that the ***number of ways of choosing r objects from n without taking into account the order*** (aka ***n choose r*** or the ***number of combinations of r objects from n***) is given by the ***binomial coefficient*** ${}^nC_r$ defined by: $${}^nC_r \equiv \binom{n}{r} \overset{def}{=} \frac{n!}{r!\,(n-r)!}$$	
Evaluate binomial coefficients such as ${}^7C_4$ and ${}^{14}C_9$ by first cancelling factorials from the numerator and denominator	
Know the results: $$\binom{n}{r} = \binom{n}{n-r}$$ $$\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$$ (***Khayyam-Pascal Identity***)	
Solve equations involving binomial coefficients such as, $$\binom{n}{n-2} = 55$$ $$\binom{n}{1} + \binom{n}{2} = 10$$ $$\binom{7}{n-1} + \binom{7}{n} = 8$$	
Prove identities involving binomial coefficients such as,	

$\binom{n}{r} + 2\binom{n}{r+1} + \binom{n}{r+2} = \binom{n+2}{r+2}$ $\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$ $\binom{n}{r}\binom{r}{s} = \binom{n}{s}\binom{n-s}{r-s}$	
Know ***Pascal's Triangle*** up to $n = 5$ and interpret the above 2 binomial coefficient results using Pascal's Triangle	
Know the ***Binomial Theorem*** : $(x + y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} \, y^r \quad (r, n \in \mathbb{W})$	
Know that the binomial theorem may also be written as: $(x + y)^n = \sum_{r=0}^{n} {}^nC_r \, x^r \, y^{n-r}$	
Know that the ***binomial expansion*** of $(x + y)^n$ is the RHS of the above equation written out fully and has $n + 1$ terms	
Expand $(x + y)^n$ for $n \leq 5$	
Expand and simplify binomial expressions such as: $(a + b)^5$ $(x + 3)^4$ $(2u - 3v)^5$ $\left(x^2 - \frac{2}{x}\right)^4$ $\left(\frac{1}{2}x - 3\right)^4$	
Know that the ***general term*** in the expansion of $(x + y)^n$ is: ${}^nC_r \, x^r \, y^{n-r}$	
Simplify the general term in expressions such as:	

$\left(x^2 + \frac{1}{x}\right)^{10}$	
Use the general term to find the coefficient of a specific term in a binomial expansion, for example: $\left(x^2 + \frac{1}{x}\right)^{10}$ (coefficient of x^{14})	
Use the general term to find a specific term in a binomial expansion, for example: $\left(x + \frac{2}{x}\right)^{9}$ (term in x^7) $\left(x - \frac{1}{x}\right)^{8}$ (term independent of x)	
Use the general term to find the specific term in a binomial expansion when told the coefficient of that term, such as: $(2 + x)^6$ (term with coefficient 64)	
Use the Binomial Theorem to calculate exactly powers such as $0 \cdot 9^4$ and $1 \cdot 02^3$	
Use the Binomial Theorem to estimate powers such as e^5 and π^3	
Know that, given events A and B with probabilities p and q satisfying $p + q = 1$ respectively, the probability of event A occurring r times and event B occurring $n - r$ times is given by, $\binom{n}{r} p^r q^{n-r}$	
Use the Binomial Theorem to solve problems involving probability such as (i) if a fair coin is flipped 7 times, calculate the probability of obtaining exactly 5 heads (ii) if the probability of rain on a given day is 0.29 calculate the probability that it will not rain in a week	

Partial Fractions

Skill	Achieved ?
Know that a ***rational function*** is of the form: $$\frac{p(x)}{q(x)}$$ where $p(x)$ and $q(x)$ are polynomials	
Know that a ***proper rational function*** is one with deg $p <$ deg q	
Know that an ***improper rational function*** is one with deg $p \geq$ deg q	
Know that any improper rational function $\frac{p}{q}$ can be written using long division as a polynomial f plus a proper rational function $\frac{g}{q}$: $$\frac{p(x)}{q(x)} = f(x) + \frac{g(x)}{q(x)}$$	
Write an improper rational function as a polynomial plus a proper rational function by long division	
Know that an ***irreducible polynomial*** is one that cannot be factorised	
Know the ***Partial Fraction Decomposition Theorem***, namely that any rational function $\frac{p}{q}$ can be written as a polynomial plus a sum of proper rational functions each of which is of the form: $$\frac{g(x)}{r(x)^n} \qquad (n \in \mathbb{N})$$ where r is an irreducible factor of q and deg $g <$ deg r; such proper rational functions are called ***partial fractions*** of $\frac{p}{q}$	
Know that if q has a non-repeated linear factor $r(x) = ax + b$, then the resulting partial fraction is of the form: $$\frac{S}{ax+b} \qquad (a \neq 0 \neq ax + b;\ S \in \mathbb{R})$$	
Know that if q has a repeated linear factor $r(x) = (ax + b)^2$, then the resulting partial fraction is of the form:	

$\frac{S}{ax+b} + \frac{T}{(ax+b)^2} \quad (a \neq 0 \neq ax+b\,;\, S, T \in \mathbb{R})$	
Know that if q has an irreducible quadratic factor $r(x) = ax^2 + bx + c \ (b^2 - 4ac < 0)$, then the resulting partial fraction is of the form: $\frac{Sx+T}{ax^2+bx+c} \quad (a \neq 0 \neq ax^2 + bx + c\,;\, S, T \in \mathbb{R})$	
Use the Partial Fraction Decomposition Theorem to find partial fractions for rational functions where the denominator is a polynomial of degree at most 3, for example: $\frac{x^3}{x^2-1}$ $\frac{x^2}{(x+1)^2}$ $\frac{1}{x^2-x-6}$ $\frac{1}{x^3+x}$ $\frac{2x^2-9x-6}{x(x^2-x-6)}$ $\frac{12x^2+20}{x(x^2+5)}$ $\frac{x^2+6x-4}{(x+2)^2(x-4)}$ $\frac{3x+5}{(x+1)(x+2)(x+3)}$ $\frac{13-x}{x^2+4x-5}$	

Differential Calculus

Skill	Achieved ?
Know the meaning of ***limit***	
Know that the derivative of a function f is defined by: $$f'(x) \overset{def}{=} \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$	
Know that ***differentiating a function from first principles*** means using the above definition explicitly	
Know the meaning of ***higher derivative***	
Know that if the first derivative of a function f is differentiable, then the ***second derivative of f*** is defined as: $$\frac{d^2 f}{dx^2} \overset{def}{=} \frac{d}{dx}\left(\frac{df}{dx}\right)$$	
Know that if f is sufficiently differentiable, then the ***n th derivative of f*** ($n \geq 2$) is defined as: $$\frac{d^n f}{dx^n} \overset{def}{=} \underbrace{\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}\cdots\left(\frac{d}{dx}\left(\frac{d}{dx}\right.\right.\right.\right.}_{n \text{ times}} f \left.\left.\left.\left.\right)\right)\cdots\right)\right)$$	
Know the ***Product Rule*** (in Euler and Lagrange notation, respectively): $$D(fg) = (Df)g + f(Dg)$$ $$(fg)' = f'g + fg'$$	
Use the product rule to differentiate functions such as: $$p(x) = x(1 + x)^{10}$$ $$y = (x + 1)(x - 2)^3$$	
Know the ***Quotient Rule*** : $$D\left(\frac{f}{g}\right) = \frac{(Df)g - f(Dg)}{g^2}$$ $$(f/g)' = \frac{1}{g^2}(f'g - fg')$$	
Use the quotient rule to differentiate functions such as:	

$y = \dfrac{x}{1 + x^2}$ $f(x) = \dfrac{x - 3}{x + 2}$ $y = \dfrac{1 + x^2}{1 + x}$ $r(x) = \dfrac{x^2 + 2x}{x^2 - 1}$	
Know the definitions of the ***reciprocal trigonometric functions*** (***secant x***, ***cosecant x*** and ***cotangent x***) : $\sec x \overset{def}{=} \dfrac{1}{\cos x}$ $\operatorname{cosec} x \overset{def}{=} \dfrac{1}{\sin x}$ $\cot x \overset{def}{=} \dfrac{\cos x}{\sin x}$	
Prove the following identities: $1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \operatorname{cosec}^2 x$	
Know that: $\operatorname{dom}(\sec x) = \mathbb{R} \setminus \left\{x \in \mathbb{R} : x = \dfrac{\pi}{2} + \pi n,\ n \in \mathbb{Z}\right\}$ $\operatorname{ran}(\sec x) = \mathbb{R} \setminus (-1, 1)$	
Know that: $\operatorname{dom}(\operatorname{cosec} x) = \mathbb{R} \setminus \{x \in \mathbb{R} : x = \pi n,\ n \in \mathbb{Z}\}$ $\operatorname{ran}(\operatorname{cosec} x) = \mathbb{R} \setminus (-1, 1)$	
Know that: $\operatorname{dom}(\cot x) = \mathbb{R} \setminus \{x \in \mathbb{R} : x = \pi n,\ n \in \mathbb{Z}\}$	

$\text{ran}(\cot x) = \mathbb{R}$	
Sketch the graphs of $y = \sec x$, $y = \text{cosec } x$ and $y = \cot x$	
Know the following derivatives: $D(\sec x) = \sec x \tan x$ $D(\text{cosec } x) = -\text{cosec } x \cot x$ $D(\cot x) = -\text{cosec}^2 x$ $D(\tan x) = \sec^2 x$	
Use the chain rule to differentiate functions such as: $f(x) = (3x^2 - 5)^2$ $g(x) = \sqrt{\cos x}$ $d(x) = \sin x°$ (answer is **not** $\cos x°$) $t(x) = \cos(\cos x)$ $k(x) = \tan 5x$ $m(x) = \text{cosec}(\sin x)$ $w(x) = \sec(6 - 9x^2)$ $p(x) = \sqrt{\cot x}$	
Use at least a double application of the chain rule to differentiate functions such as: $r(x) = \dfrac{1}{\sin^2(4x + 2)}$ $n(x) = \cos\left(\dfrac{1}{x^2 + 6x + 9}\right)$	
Know the definition of the ***natural logarithm function*** as: $\ln x \overset{def}{=} \displaystyle\int_1^x \frac{1}{t}\, dt$	

Know that the derivative of the natural logarithm function is: $$D(\ln x) = \frac{1}{x}$$	
Know the definition of the ***exponential function to base e*** as the inverse of the natural logarithm function: $$\exp \overset{def}{=} \ln^{-1}$$ or, in terms of composition: $$e^{\ln x} = \ln e^{x} = x$$	
Know that the derivative of the exponential function to base *e* is: $$D(e^{x}) = e^{x}$$ sometimes written as: $$D(\exp x) = \exp x$$	
Use the chain rule to differentiate functions such as: $$k(x) = \exp(7x)$$ $$g(x) = \ln(8x)$$ $$t(x) = e^{5x^3+4}$$ $$j(x) = \ln(3x^2 - 4)$$ $$v(x) = e^{e^{x}}$$ $$b(x) = \ln(\sec x)$$ $$d(x) = \exp(\sin 2x)$$ $$p(x) = e^{\cot 2x}$$ $$f(x) = \ln(\ln x)$$ $$r(x) = e^{\ln 6x}$$ $$s(x) = \ln(\cos 2x)$$	

$f(x) = \sqrt{\sin x}$	
Use a combination of the product, quotient and chain rules to differentiate functions such as: $f(x) = \sqrt{x}\, e^{-x}$; $v(x) = \cos^2 x\, e^{\tan x}$; $e(x) = x^3 \tan 2x$; $y = \dfrac{1 + \ln x}{3x}$; $s(x) = \dfrac{x}{\ln 7x}$; $t(x) = e^x \sin x^2$; $a(x) = \dfrac{x^3}{(1 + \tan x)}$; $b(x) = \dfrac{\cot x + \sec x}{\cot x - \sec x}$	
Find the second derivative of a function, for example: $f(x) = \dfrac{x}{\ln x}$	

Applications of Differentiation

Skill	Achieved ?
Know that ***rectilinear motion*** means motion in a straight line (or along an axis, usually the x – axis)	
Know that ***displacement (from the origin)*** is a function of time $s\ (t)$	
Know that ***velocity*** is the first derivative of displacement: $$v\ (t) \stackrel{def}{=} \frac{ds}{dt}$$	
Know that ***acceleration*** is the first derivative of velocity (equivalently, the second derivative of displacement): $$a\ (t) \stackrel{def}{=} \frac{dv}{dt} = \frac{d^2s}{dt^2}$$	
Know the ***Newton notation*** (aka ***dot notation***) for derivatives (especially derivatives with respect to time): $$\dot{s}\ (t) \stackrel{def}{=} \frac{ds}{dt}$$	
Know that velocity and acceleration can be written in Newton notation as, respectively: $$v\ (t) = \dot{s}\ (t)$$ and $$a\ (t) = \ddot{s}\ (t)$$	
Given displacement, calculate velocity	
Given displacement or velocity, calculate acceleration	
Know that the ***right-hand derivative of f at x = a*** is: $$f_+{}'(a) \stackrel{def}{=} \lim_{h \to 0^+} \left(\frac{f(a + h) - f(a)}{h} \right)$$	
Know that the ***left-hand derivative of f at x = a*** is: $$f_-{}'(a) \stackrel{def}{=} \lim_{h \to 0^-} \left(\frac{f(a + h) - f(a)}{h} \right)$$	
Know that a function is differentiable at $x = a$ if the left-hand derivative at $x = a$ exists, the right-hand derivative at $x = a$ exists and these 2 derivatives have the same value at a	
Know that a function is not differentiable at $x = a$ if either	

$\nexists f_+'(a)$ or $\nexists f_-'(a)$ or $f_+'(a) \neq f_-'(a)$	
Know that the non-differentiability of a function can be discerned from its graph by identifying 'sharp corners'	
Know that a function f has a ***critical point*** at $x = a$ if either (i) $f'(a) = 0$ or (ii) $\nexists f'(a)$	
Know that if $(a, f(a))$ is a critical point, then a is called a ***critical number*** and $f(a)$ a ***critical value***	
Find critical points of functions	
Know that a function can have 3 types of extrema: ***Local extrema*** ***Endpoint extrema*** ***Global extrema***	
Know that a function f has a ***local (relative) maximum at $x = a$*** if $\exists$ an open interval I about a for which $f(a) \geq f(x)\ \forall x \in I$	
Know that a function has a ***local (relative) minimum at $x = a$*** if $\exists$ an open interval I about a for which $f(a) \leq f(x)\ \forall x \in I$	
Know that all local extrema occur at critical points	
Find local extrema of functions	
Know that not all critical points are local extrema	
Know that, if a is an endpoint in dom f, f has an ***endpoint maximum at $x = a$*** if $\exists\ p \in$ dom f for which $f(a) \geq f(x)$ $(\forall x \in [a, p)$ or $(p, a])$	
Know that, if a is an endpoint in dom f, f has an ***endpoint minimum at $x = a$*** if $\exists\ p \in$ dom f for which $f(a) \leq f(x)$ $(\forall x \in [a, p)$ or $(p, a])$	
Know that a function has a ***global (absolute) maximum at $x = a$*** if $f(a) \geq f(x)$ $(\forall x \in$ dom $f)$	
Know that a function has a ***global (absolute) minimum at $x = a$*** if $f(a) \leq f(x)$ $(\forall x \in$ dom $f)$	
Know that every global extremum is either a local extremum or an endpoint extremum	
Find global extrema of functions	
Know the ***second derivative*** test for local maxima and local minima: If $f'(a) = 0$ and $f''(a) > 0$, then f has a local minimum at a If $f'(a) = 0$ and $f''(a) < 0$, then f has a local maximum at a	
Use the second derivative test to find local extrema	
Know that function is ***concave up*** on an interval if $f''(x) > 0$	

AH Checklist (Unit 1)

Know that function is ***concave down*** on an interval if $f''(x) < 0$	
Know that a P of I occurs at a point when there is a change of concavity as the graph passes through that point	
Know that if f has a point of inflexion at $x = a$, then either $f''(a) = 0$ or $\nexists\ f''(a)$	
Know that a function may have a ***non-horizontal point of inflexion***, for example: $f(x) = \dfrac{x}{\ln x}$	
Find the coordinates and nature of stationary points of functions such as: $f(x) = \dfrac{x^2 + 6x + 12}{x + 2}$ $y = \dfrac{x^2}{(x + 1)^2}$ $y = \dfrac{x}{1 + x^2}$ $y = \dfrac{x^3}{x - 2}$	
Prove that a function has no stationary points, for example: $f(x) = \dfrac{x - 3}{x + 2}$	
Prove that a function has no points of inflexion, for example: $f(x) = \dfrac{x - 3}{x + 2}$	
Solve optimisation problems (which may involve critical points)	

Integral Calculus

Skill	Achieved ?
Know the standard integrals: $\int e^x \, dx = e^x + C$ $\int \frac{1}{x} \, dx = \ln\lvert x\rvert + C$ $\int \sec^2 x \, dx = \tan x + C$	
Know the technique of ***integration by substitution***	
Know certain general forms of substitution (in both cases, letting $u = f(x)$) : $\int (Df)\, f \, dx = \frac{1}{2}f^2 + C$ $\int \frac{Df}{f} \, dx = \ln\lvert f\rvert + C$	
Use integration by substitution to find indefinite integrals such as: $\int \frac{1}{x^2 + 4x + 8} \, dx \qquad (x + 2 = 2\tan\theta)$ $\int \frac{1}{(1 + \sqrt{x})^3} \, dx \qquad (x = (u - 1)^2)$ $\int \frac{x^3}{1 + x^8} \, dx \qquad (t = x^4)$ $\int \frac{x}{\sqrt{1 - x^2}} \, dx \qquad (u = 1 - x^2)$ $\int \frac{12x^3 - 6x}{x^4 - x^2 + 1} \, dx$	

$\int \frac{x}{\sqrt{1 - 49x^4}} \, dx$	
Use integration by substitution to find definite integrals such as: $\int_0^{\pi/2} \frac{\cos\theta}{(1 + \sin\theta)^3} \, d\theta \quad (x = 1 + \sin\theta)$ $\int_0^3 \frac{x}{\sqrt{1 + x}} \, dx \quad (u = 1 + x)$ $\int_0^1 \frac{x^3}{(1 + x^2)^4} \, dx \quad (u = 1 + x^2)$ $\int_{\ln 3/2}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx$ $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}} \, dx \quad (x = 2\sin\theta)$	
Calculate the area between a function, the y – axis and 2 points c and d on the y – axis using the formula: $A = \int_c^d f^{-1}(y) \, dy$	
Calculate the area between 2 curves $x = f^{-1}(y)$ and $x = g^{-1}(y)$ (assuming $y = f(x)$ and $y = g(x)$ are invertible) by integrating 'right-hand function – left-hand function' between two limits c and d, $A = \int_c^d (f^{-1}(y) - g^{-1}(y)) \, dy = \int_c^d f^{-1}(y) \, dy - \int_c^d g^{-1}(y) \, dy$ where $f^{-1}(y) \geq g^{-1}(y)$ and $c \leq y \leq d$	
Know the meaning of ***solid of revolution***	
Know the meaning of ***volume of solid of revolution***	

Know that the volume of solid of revolution formed by rotating the graph of the function $y = f(x)$ 360° about the x - axis between $x = a$ and $x = b$ is given by: $$V = \pi \int_a^b y^2 \, dx$$	
Know that the volume of solid of revolution formed by rotating the graph of the function $x = g(y)$ 360° about the y - axis between $y = c$ and $y = d$ is given by: $$V = \pi \int_c^d x^2 \, dy$$	
Calculate volumes of solids of revolution, such as: $y = e^{-2x}$ between $x = 0$ and $x = 1$, 360° about the x – axis $y = \dfrac{x^{3/2}}{(1 + x^2)^2}$ between $x = 0$ and $x = 1$, 360° about the x – axis Region between $y = x^2$ and $y^2 = 8x$, 360° about the y – axis	
Know that velocity is the integral of acceleration and that displacement is the integral of velocity: $$v(t) = \int a(t) \, dt$$ $$s(t) = \int v(t) \, dt$$	
Know that a useful visualisation (and calculation) aid in linking displacement, velocity and acceleration is: $$s(t) \underset{\int}{\overset{\frac{d}{dx}}{\rightleftarrows}} v(t) \underset{\int}{\overset{\frac{d}{dx}}{\rightleftarrows}} a(t)$$	
Know that ***at rest (when $t = a$)*** means $v(a) = 0$; usually $a = 0$	

Properties of Functions

Skill	Achieved ?
Know the definition of the ***modulus function*** : $$\lvert x \rvert = \begin{cases} x & (x \geq 0) \\ -x & (x < 0) \end{cases}$$	
Know that, for example, $\lvert 7 \rvert = 7$, $\lvert 0 \rvert = 0$ and $\lvert -3 \rvert = 3$	
Know that the ***modulus of a function f*** is given by: $$\lvert f \rvert = \begin{cases} f & (f \geq 0) \\ -f & (f < 0) \end{cases}$$	
Given the graph of a function, sketch the graph of the modulus of that function, for example: $$\lvert x^2 - 3 \rvert$$ $$\lvert \sin x \rvert$$ $$\lvert \cos 2x \rvert$$ $$\lvert \ln x \rvert$$ $$\lvert x^3 + 2 \rvert$$ $$\lvert \sec x \rvert$$	
Know the definitions of the ***inverse trigonometric functions*** - ***inverse sine*** (aka ***arcsine***, written $\sin^{-1}$), ***inverse cosine*** (aka ***arccosine***, written $\cos^{-1}$) and ***inverse tangent*** (aka ***arctangent***, written $\tan^{-1}$) as the inverse of the sine, cosine and tangent functions	
Know that: $$\text{dom}\,(\sin^{-1} x) = [\,-1, 1]$$ $$\text{ran}\,(\sin^{-1} x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$	
Know that:	

dom $(\cos^{-1} x) = [-1, 1]$ ran $(\cos^{-1} x) = [0, \pi]$	
Know that: dom $(\tan^{-1} x) = \mathbb{R}$ ran $(\tan^{-1} x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	
Sketch the graphs of $y = \sin^{-1} x$, $y = \cos^{-1} x$ and $y = \tan^{-1} x$	
Know that f is an ***even function*** if: $f(x) = f(-x) \quad (\forall x \in \text{dom } f)$	
Know that f is an ***odd function*** if: $f(x) = -f(x) \quad (\forall x \in \text{dom } f)$	
Sketch the graph of an even or odd function	
Know that a function is ***neither even nor odd*** if it is not even and if it is not odd	
Determine whether a given function is even, odd or neither, such as: $w(x) = x^4 - 8x^2 + 7$ $p(x) = x^3 - 4x$ $m(x) = x^2 + x$ $f(x) = x^4 \sin 2x$ $g(x) = x^2 \cos 3x$ $h(x) = x^3 \tan 4x$ $n(x) = e^x - e^{-x}$ $a(x) = e^x + e^{-x}$ $d(x) = x + \frac{1}{x}$	
Given the graph of a function, determine whether the function is even, odd or neither	
Know that asymptotes can be of 3 types:	

AH Checklist (Unit 1)

Vertical Asymptotes (Equation: x = constant) ***Horizontal Asymptotes*** (Equation: y = constant) ***Oblique Asymptotes*** (Equation: $y = mx + c$)	
Recall that any improper rational function $\frac{p}{q}$ can be written by long division as a polynomial f plus a proper rational function $\frac{g}{q}$: $$\frac{p(x)}{q(x)} = f(x) + \frac{g(x)}{q(x)}$$	
Know that the solutions of $q(x) = 0$ give vertical asymptotes and that either: (i) $y = f(x)$ is a horizontal asymptote if f is a constant function or (ii) $y = f(x)$ is an oblique asymptote if f is a linear function	
Find or state asymptotes for rational functions	
Sketch the graph of a rational function, indicating stationary points, asymptotes and intersections with axes, for example: $$f(x) = \frac{x^2 + 6x + 12}{x + 2}$$ $$g(x) = \frac{x^2 + 2x}{x^2 - 1}$$ $$r(x) = \frac{x^2}{(x + 1)^2}$$	
Given the graph of a function, sketch the graph of a closely related modulus function, indicating the new critical points, such as: Given $y = \frac{x}{1 + x^2}$, sketch $y = \left\|\frac{x}{1 + x^2}\right\|$ Given $y = \frac{x^3}{x - 2}$, sketch $y = \left\|\frac{x^3}{x - 2}\right\| + 1$	
Given the graph of a function with asymptotes, sketch the graph of a related function indicating the new asymptotes	

AH Checklist (Unit 1)

Systems of Equations and Gaussian Elimination

Skill	Achieved ?
Know that a 3×3 system of (linear) equations is of the form: $$\begin{aligned} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{aligned}$$	
Know that the ***coefficient matrix*** for the above system is: $$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$	
Know that the ***Augmented Matrix*** for the above system is: $$\left(\begin{array}{ccc\|c} a & b & c & j \\ d & e & f & k \\ g & h & i & l \end{array}\right)$$	
Know that a system of equations can be solved by applying to the Augmented Matrix ***elementary row operations (EROs)***, which are of are of 3 types: Interchanging 2 or more rows Multiplying a row by a non-zero real number Replacing a row by adding it to a multiple of another row	
Know that EROs do not change the solution of a system of equations	
Know the meaning of ***row reduction*** and ***row reduce***	
Know that an augmented matrix (or coefficient matrix) is in ***row echelon form*** if each non-zero row has more ***leading zeros*** than the previous row: $$\left(\begin{array}{ccc\|c} a & b & c & j \\ 0 & e & f & k \\ 0 & 0 & i & l \end{array}\right)$$	
Use EROs to row reduce a system of equations into row echelon form	
Know that ***Gaussian Elimination*** is the technique of row reducing a system of equations to row echelon form	

Know that a system of equations has either: ***No solution*** (aka ***inconsistency***) ***A unique solution*** ***Infinitely many solutions***	
Know that a system with no solutions is characterised by a row reduced matrix of the form: $\left(\begin{array}{ccc\|c} a & b & c & j \\ 0 & e & f & k \\ 0 & 0 & 0 & l \end{array}\right) \quad (l \neq 0)$	
Know that a system with a unique solution is characterised by a row reduced matrix of the form: $\left(\begin{array}{ccc\|c} a & b & c & j \\ 0 & e & f & k \\ 0 & 0 & i & l \end{array}\right) \quad (i \neq 0)$	
Know that a system with infinitely many solutions is characterised by a row reduced matrix of the form: $\left(\begin{array}{ccc\|c} a & b & c & j \\ 0 & e & f & k \\ 0 & 0 & 0 & 0 \end{array}\right)$	
Know that a system with infinitely many solutions has a ***free variable*** (aka ***parameter***), say $z = t$, and then the solution set is expressed in terms of this parameter	
Know the meaning of ***back-substitution***	
Use back-substitution to solve a system of equations	
Use Gaussian Elimination to solve systems of equations such as: $\begin{array}{rcrcrcr} x & + & y & + & z & = & 10 \\ 2x & - & y & + & 3z & = & 4 \\ x & & & + & 2z & = & 20 \end{array}$ $\begin{array}{rcrcrcr} x & + & y & + & 3z & = & 2 \\ 2x & + & y & + & z & = & 2 \\ 3x & + & 2y & + & 5z & = & 5 \end{array}$	

$x + y + 3z = 1$ $3x + ay + z = 1$ $x + y + z = -1$ $x + y + 2z = 1$ $2x + by + z = 0$ $3x + 3y + 9z = 5$ $2x - y + 2z = 1$ $x + y - 2z = 2$ $x - 2y + 4z = -1$ $x + y - z = 6$ $2x - 3y + 2z = 2$ $-5x + 2y - 4z = 1$ $x - y + z = 1$ $x + y + 2z = 0$ $2x - y + az = 2$	
Know that a system of equations is ***ill-conditioned*** when changing the entries of the Augmented Matrix induce a large change in the solution set of the system of equations	
Know the geometric interpretation of a system of 2 equations in 2 variables to be ill-conditioned, namely, that the lines representing each equation are almost parallel	
Determine whether or not a system of 2 equations in 2 variables is ill-conditioned	