Matrices

Skill	Achieved ?
Know that a ***matrix*** is a rectangular array of numbers (aka ***entries*** or ***elements***) in parentheses, each entry being in a particular ***row*** and ***column***	
Know that the ***order*** of a matrix is given as $m \times n$ (read ***m by n***), where m is the number of rows and n the number of columns and is written as: $$A \equiv (a_{ij})_{m\times n} \overset{def}{=} \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{pmatrix}$$	
Know that the ***main diagonal*** (aka ***leading diagonal***) of any matrix is the set of entries a_{ij} where $i = j$	
Know that the element in row i and column j of a matrix is written as a_{ij} and called the ***(i, j)*** th ***entry of A***	
Know that a ***row matrix*** is a $1 \times n$ matrix and is written as: $$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} & a_{1n} \end{pmatrix}$$	
Know that a ***column matrix*** is a $m \times 1$ matrix and is written as: $$\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{(m-1)1} \\ a_{m1} \end{pmatrix}$$	
Know that a ***square matrix (of order m*** $\times$ ***m*** or just ***of order m)*** is a matrix with the same number of rows as columns (equal to m) and is written as: $$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \\ a_{m1} & a_{m2} & & a_{mm} \end{pmatrix}$$	
Know that the ***identity matrix (of order m)*** is the $m \times m$ matrix all of whose entries are 0 apart from those on the	

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main diagonal, where they all equal 1: $$I_m \overset{def}{=} \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 1 \end{pmatrix}$$	
Know that the ***zero matrix (of order m × n)*** is the $m \times n$ matrix all of whose entries are 0: $$O_{m \times n} \overset{def}{=} \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$	
Know that the ***transpose*** of an $m \times n$ matrix A (denoted A^{T}) is the $n \times m$ matrix obtained by interchanging the rows and columns of A ($(a^{\mathsf{T}})_{ij}$ denotes the $(i, j)^{th}$ entry of A^{T}): $$(a^{\mathsf{T}})_{ij} \overset{def}{=} a_{ji}$$	
Obtain the transpose of any matrix	
Know that 2 matrices are equal if they both have the same order and all corresponding entries are equal	
Know that matrices can be added or subtracted only if they have the same order	
Add or subtract 2 matrices A and B (thus obtaining the ***matrix sum*** $A + B$ or ***matrix difference*** $A - B$) by adding or subtracting the corresponding entries of each matrix ($(a \pm b)_{ij}$ denotes the $(i, j)^{th}$ entry of the sum or difference): $$(a \pm b)_{ij} \overset{def}{=} a_{ij} \pm b_{ij}$$	
Add or subtract more than 2 matrices	
Multiply any matrix A by a real scalar k (thus obtaining the ***scalar multiplication*** kA) by multiplying each entry of A by k ($(ka)_{ij}$ denotes the $(i, j)^{th}$ entry of kA): $$(ka)_{ij} \overset{def}{=} ka_{ij} \quad (k \in \mathbb{R})$$	
Scalar multiply a matrix by a real scalar	
Know, use and verify the following matrix properties (for $k \in \mathbb{R}$):	

$A + B = B + A$ $(A + B) + C = A + (B + C)$ $k(A + B) = kA + kB$ $(A + B)^{T} = A^{T} + B^{T}$ $(A^{T})^{T} = A$ $(kA)^{T} = kA^{T}$	
Know that 2 matrices A and B can only be multiplied in the order A times B (thus obtaining the ***matrix product*** AB) if the number of columns of A equals the number of rows of B	
Multiply 2 matrices A (of order $m \times n$) and B (of order $n \times p$) according to the rule, where $(ab)_{ij}$ denotes the $(i, j)^{th}$ entry of AB: $(ab)_{ij} \stackrel{def}{=} \sum_{k=1}^{n} a_{ik} b_{kj}$ $(1 \leq i \leq m$ and $1 \leq j \leq p)$	
Know that, in general: $AB \neq BA$	
Know that when forming AB, we say that ***B is pre-multiplied by A***, or ***A is post-multiplied by B***	
Multiply 2 or more matrices where m and n are at most equal to 3	
Know that a (square) matrix A can be multiplied by itself any number of times (thus obtaining ***the n*** th ***power of A***): $A^{n} \stackrel{def}{=} \underbrace{A \times A \times A \times \ldots \times A}_{n \text{ times}}$	
Know, use and verify the following matrix properties: $A(BC) = (AB)C$ $A(B + C) = AB + AC$ $(AB)^{T} = B^{T}A^{T}$	
Know that a matrix A is ***symmetric*** if $A^{T} = A$ (thus, A is square)	
Know that a symmetric matrix is symmetrical about the main diagonal	

Know that a matrix A is ***skew-symmetric*** (aka ***anti-symmetric***) if $A^{\mathrm{T}} = -A$ (thus, A is square)	
Know that a skew-symmetric matrix has all main diagonal entries equal to 0	
Know that a square matrix A (of order $n \times n$) is ***orthogonal*** if: $$A^{\mathrm{T}} A = I_n$$	
Know that a system of m equations in n variables may be written in matrix form, where A is the $m \times n$ ***matrix of coefficients*** (aka ***coefficient matrix***), $\mathbf{x}$ is the $n \times 1$ ***solution vector*** and $\mathbf{b}$ an $m \times 1$ column vector as: $$A\mathbf{x} = \mathbf{b}$$	
Know that for an $n \times n$ matrix A, the ***minor*** of entry a_{ij} is the determinant (denoted M_{ij}) of the $(n - 1) \times (n - 1)$ matrix formed from A by deleting the i^{th} row and j^{th} column of A	
Know that the ***cofactor*** of entry a_{ij} is the quantity: $$C_{ij} \overset{def}{=} (-1)^{i+j} M_{ij}$$	
Know that for the ***determinant*** of a general $n \times n$ matrix is given by the ***Laplace expansion formula***: $$\det(A) \equiv \lvert A \rvert \overset{def}{=} \sum_{j=1}^{n} a_{ij} C_{ij} \quad (i = 1, 2, 3, \ldots, n)$$	
Know that a system of n equations in n unknowns has a solution if the determinant of the coefficient matrix is non-zero	
Know that the determinant of a 1×1 matrix is: $$\lvert A \rvert \equiv \lvert (a) \rvert = a$$	
Know that the determinant of a 2×2 matrix is: $$\lvert A \rvert \equiv \left\lvert \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\rvert = ad - bc$$	
Calculate the determinant of any 2×2 matrix	
Know that the determinant of a 3×3 matrix is: $$\lvert A \rvert \equiv \left\lvert \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \right\rvert = a \left\lvert \begin{pmatrix} e & f \\ h & i \end{pmatrix} \right\rvert - b \left\lvert \begin{pmatrix} d & f \\ g & i \end{pmatrix} \right\rvert + c \left\lvert \begin{pmatrix} d & e \\ g & h \end{pmatrix} \right\rvert$$	

Calculate the determinant of any 3 $\times$ 3 matrix	
Know that an $n \times n$ matrix A has an ***inverse*** if there is a matrix (denoted A^{-1}) such that: $AA^{-1} = I_n$ or $A^{-1}A = I_n$	
Know that there is only 1 inverse (if it exists) for any given matrix	
Know that a matrix is ***invertible*** (aka ***non-singular***) if it has an inverse	
Know that if a matrix does not have an inverse, then it is said to be ***non-invertible*** (aka ***singular***)	
Know that a matrix is invertible iff det $(A) \neq 0$	
Know that a matrix is non-invertible iff det $(A) = 0$	
Given a missing variable in a 2 $\times$ 2 or 3 $\times$ 3 matrix, obtain the value(s) of this variable for which the matrix is singular or non-singular	
Know that the ***cofactor matrix*** of square matrix A is the matrix C whose $(i, j)^{th}$ entry is C_{ij}	
Know that the ***adjugate*** (aka ***classical adjoint***) of a square matrix A is the transpose of the cofactor matrix C: $\text{adj}(A) \stackrel{def}{=} C^{\mathsf{T}}$	
Know that the inverse of a matrix A is given by: $A^{-1} = \dfrac{\text{adj}(A)}{\det(A)}$	
Calculate the inverse of a 2 x 2 matrix using the formula: $A^{-1} = \dfrac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	
Know, use and verify the following matrix properties: $\lvert AB \rvert = \lvert A \rvert \times \lvert B \rvert$; $\lvert kA \rvert = k^n \lvert A \rvert$ $(k \in \mathbb{R}, n \in \mathbb{N}, A$ is $n \times n)$; $\lvert A^{\mathsf{T}} \rvert = \lvert A \rvert$; $\lvert A^{-1} \rvert = \dfrac{1}{\lvert A \rvert}$	

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$(A^{-1})^{-1} = A$ $(A^{-1})^{T} = (A^{T})^{-1}$ $(kA^{-1}) = \frac{1}{k} A^{-1}$ $(A\,B)^{-1} = B^{-1} A^{-1}$	
Calculate the inverse of a 3 x 3 matrix using EROs by forming a big Augmented matrix consisting of A on the LHS and the identity matrix on the right, then row reducing the Augmented Matrix until the identity is reached on the left and whatever remains on the RHS is A^{-1}	
Know that a system of equations $A\mathbf{x} = \mathbf{b}$ can be solved by premultiplying each side of this equation by A^{-1}, and so the solution vector is obtained as: $\mathbf{x} = A^{-1}\mathbf{b}$	
Know that a ***linear transformation in the plane*** is a function that sends a point P(x, y) to a point Q($ax + by$, $cx + dy$) for $a, b, c, d \in \mathbb{R}$	
Know that a linear transformation in the plane can be described as a matrix equation: $\begin{pmatrix} x' \\ y' \end{pmatrix} \overset{def}{=} \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$	
Know that in the above equation, the matrix with entries a, b, c and d is called the ***transformation matrix***	
Know that if a transformation is represented by a matrix A, then the reverse of that transformation is represented by A^{-1}	
Know that a transformation matrix can be obtained by considering the effect of a geometrical transformation on the points (0, 1) and (1, 0)	
Know that an ***invariant point*** is one that has the same image under a transformation	
Find invariant points of a given transformation by solving the equation: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$	
Know that a ***reflection in the line $y = x$***	

has transformation matrix: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	
Know that an ***anticlockwise rotation of angle θ (about the origin)*** has transformation matrix: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	
Know that a ***dilatation*** (aka ***scaling***) has transformation matrix: $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ $(k \in \mathbb{R})$	
Derive transformation matrices for other geometrical transformations, for example: Reflection in the line $y = -x$ Reflection in the x – axis Reflection in the y – axis Half-turn anticlockwise about the origin Quarter-turn clockwise about the origin	
Know that a combination of transformations is found by matrix multiplication	
Derive the transformation matrix of a combination of transformations, and describe the effect this combination has on a given point, for example: reflection in the x – axis, then anti-clockwise rotation of 30° anti-clockwise rotation of $\frac{\pi}{2}$ radians, then reflection in the x – axis enlargement of scale factor 2, then a clockwise rotation of 60°	
Find the equation of the image of a given curve (possibly giving the answer in implicit form) under a given transformation	

Vectors, Planes and Lines

Skill	Achieved ?
Know that the ***direction ratio*** of a vector is the ratio of its components in the order $x : y\ (: z)$	
Determine the direction ratio of a vector	
Know that 2 vectors with the same direction ratio are parallel	
Know that if α, β and γ are the angles the vector **v** makes with the x, y and z axes, and **u** is a unit vector in the direction of **v**, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the ***direction cosines*** of **v** and: $\mathbf{u} = \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$	
Determine the direction cosines of a vector	
Know that the ***vector product*** (aka ***cross product***) of 2 vectors **a** and **b**, where θ is the angle from **a** to **b**, and **n** is a unit vector at right angles to both **a** and **b**, is defined as: $\mathbf{a} \times \mathbf{b} \overset{def}{=} \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert \sin\theta\ \mathbf{n}$	
Know that the vector product is only defined in 3D and $\mathbf{a} \times \mathbf{b}$ is a vector at right angles to both **a** and **b**	
Know that the vector product is a vector, not a scalar	
Know that the 3 unit vectors, **i**, **j** and **k** satisfy: $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$ and $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$	
Know the following properties of the vector product: $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ $\mathbf{a} \times \mathbf{b} = -\,\mathbf{b} \times \mathbf{a}$	

$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$ $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \times \mathbf{c}) + (\mathbf{b} \times \mathbf{c})$	
Know that if the vector product of 2 vectors is **0**, then they are parallel	
Know that if 2 non-zero vectors are parallel, then their vector product is **0**	
Given the components of 2 vectors, calculate their vector product using the ***component form of the vector product***: $\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\,\mathbf{i} + (a_3b_1 - a_1b_3)\,\mathbf{j} + (a_1b_2 - a_2b_1)\,\mathbf{k}$	
Given the magnitude of 2 vectors and the angle between them, calculate their vector product	
Given 2 vectors in component form and the angle between them, calculate their vector product	
Given 2 vectors in component form, calculate their vector product	
Calculate the ***scalar triple product*** of 3 vectors using: $[\mathbf{a}, \mathbf{b}, \mathbf{c}] \equiv \mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) \stackrel{def}{=} \left\| \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \right\|$	
Know that a ***Cartesian equation of a plane*** containing a point P(x, y, z) is: $ax + by + cz = d \qquad (a, b, c, d, x, y, z \in \mathbb{R})$	
Know that the equation of a plane is not unique	
Determine whether or not a point lies on a plane	
Know that a vector is ***parallel*** to a plane if it lies in the plane	
Know that a ***normal vector*** is at right angles to any vector in the plane and is given by: $\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$	
Know that parallel planes have the same direction ratios for their normals	
Know that 2 planes are coincident if they have the same equation (possibly after simplifying one of the equations)	
Know that the angle between 2 planes is defined to be the acute angle between their normals	
Given 2 planes, calculate the angle between them	

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Find an equation for a plane, given 3 points in the plane	
Find an equation for a plane, given 2 vectors in the plane	
Find an equation for a plane given 1 point on the plane and a normal to the plane	
Calculate the distance between 2 planes	
Know that, for a point A (with position vector **a**) in a plane, and 2 vectors **b** and **c** parallel to the plane, a ***vector equation (aka parametric equation) of a plane*** for a point R (with position vector **r**), where t and u are real parameters, is: $\mathbf{r} = \mathbf{a} + t\mathbf{b} + u\mathbf{c}$	
Find a vector equation for a plane	
Convert between the 2 different types of equations for a plane	
Know that a ***vector equation for a line*** in 3D with ***direction vector*** $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ passing through a point A(x_1, y_1, z_1) and a general point P(x, y, z) is: $\mathbf{p} = \mathbf{a} + t\mathbf{u} \quad (t \in \mathbb{R})$	
Know that the above equation can be written in ***parametric form*** as: $x = x_1 + at, \quad y = y_1 + bt, \quad z = z_1 + ct$	
Know that the above equation can be written in ***symmetric form*** (aka ***standard form*** or ***canonical form***), provided that none of a, b or c equal 0, as: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = t$	
Convert, where possible, between the 3 forms of a line equation	
Know that a line equation is not unique	
Verify that a point lies on a line	
Given 2 points on a line, determine an equation for L	
Know that 2 lines are parallel if the direction ratios of their direction vectors are equal	
Know that 2 lines are coincident if they have the same equation (possibly after simplifying one of the equations)	
Given a point lying on a line L and a direction vector for L, determine an equation for L	
Given a point lying on a line L, and 2 vectors that are perpendicular to the direction of L, obtain the equation of L	
Know that non-coincident lines in 3D may (i) be parallel (ii) intersect at a point or (iii) be ***skew*** (neither parallel nor intersecting)	

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Given 2 lines, determine whether or not they intersect	
Given 2 lines (either both in parametric form, both in symmetric form, or one in parametric form and the other in symmetric form) that intersect, find their point of intersection	
Given the equations of 2 lines, calculate the size of the acute angle between them	
Calculate the shortest distance between 2 straight lines	
Given a line and a point on a plane that is perpendicular to the line, determine an equation of the plane	
Calculate the angle between a line and a plane	
Calculate the intersection point of a line and a plane	
Determine the shortest distance between a point on a plane and a line perpendicular to a normal of the plane	
Know that 2 non-coincident planes may either (i) not intersect or (ii) intersect in a straight line	
Given 2 planes, determine whether or not they intersect	
Given 2 intersecting planes, find their line of intersection	
Know that the intersection of 3 planes can be modelled by a system of 3 equations in 3 variables and the possibilities are: Unique intersection point 1 line of intersection 2 parallel lines of intersection 3 parallel lines of intersection 1 plane of intersection no intersection	
Find the intersection of 3 planes	

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Further Sequences and Series

Skill	Achieved ?
Know ***d'Alembert's ratio test*** (aka ***ratio test***) for deciding whether a series of positive terms, $\sum_{n=1}^{\infty} u_n$, converges or diverges: $\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right) < 1 \Rightarrow$ series converges; $\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right) > 1 \Rightarrow$ series diverges; $\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right) = 1 \Rightarrow$ no conclusion	
Know that a series (not necessarily one with all terms positive) $\sum_{n=1}^{\infty} u_n$ is ***absolutely convergent*** if $\sum_{n=1}^{\infty} \lvert u_n \rvert$ converges	
Know that any absolutely convergent series is convergent	
Know that if $\sum_{n=1}^{\infty} u_n$ converges but $\sum_{n=1}^{\infty} \lvert u_n \rvert$ diverges, then $\sum_{n=1}^{\infty} u_n$ is said to be ***conditionally convergent***	
Know that any rearrangement of an absolutely convergent series converges to the same limit	
Know the ***Riemann rearrangement theorem***, namely, that any conditionally convergent series can be rearranged to converge to any real number, or rearranged to diverge	
Know that for absolute convergence, d'Alembert's ratio test is: $\lim_{n\to\infty}\left\lvert\frac{u_{n+1}}{u_n}\right\rvert < 1 \Rightarrow$ series converges absolutely; $\lim_{n\to\infty}\left\lvert\frac{u_{n+1}}{u_n}\right\rvert > 1 \Rightarrow$ series diverges	

$\lim_{n\to\infty}\left\|\frac{u_{n+1}}{u_n}\right\| = 1 \Rightarrow$ no conclusion	
Know that the x values for which a power series converges is called the ***interval of convergence***	
Know that d'Alembert's ratio test for a power series $\sum_{n=0}^{\infty} a_n x^n$ is: $\lim_{n\to\infty}\left\|\frac{a_{n+1}\,x}{a_n}\right\| < 1 \Rightarrow \sum_{n=0}^{\infty} a_n x^n$ converges absolutely $\lim_{n\to\infty}\left\|\frac{a_{n+1}\,x}{a_n}\right\| > 1 \Rightarrow \sum_{n=0}^{\infty} a_n x^n$ diverges $\lim_{n\to\infty}\left\|\frac{a_{n+1}\,x}{a_n}\right\| = 1 \Rightarrow$ no conclusion	
Find S_∞ for power series (stating the interval of convergence), for example: $S_n = 1 + 3x + 5x^2 + 7x^3 + \ldots + (2n - 1)x^{n-1} + \ldots$ $S_n = 1 + 3x + 7x^2 + 15x^3 + \ldots$	
Know that the ***Maclaurin Series*** (aka ***Maclaurin Expansion***) of a function f is the power series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$	
Know that a truncated Maclaurin series can be viewed as an approximation of a function by a polynomial	
Write down or find the Maclaurin series for: e^x $\sin x$ $\cos x$ $\tan^{-1} x$ $\ln(1 + x)$	

Know the range of validity of the above 5 Maclaurin series	
Given the Maclaurin series for $f(x)$, obtain the Maclaurin series for $f(ax)$ ($a \in \mathbb{R} \setminus \{0\}$), for example: e^{2x} $\sin 3x$	
Evaluate Maclaurin series that are a combination of the above, for example: $(2 + x)\ln(2 + x)$ (first 4 terms) $\ln(\cos x)$ (up to the term in x^4) $\sin^2 x$ (up to the term in x^4) $\cos^2 x$ (up to the term in x^4) $e^x \sin x$ (first 3 non-zero terms) e^{x+x^2} (up to the term in x^4) $\frac{1}{2}\cos 2x$ (up to the term in x^4) $\frac{1}{2}\cos 6x$ (first 3 non-zero terms) $x\ln(2 + x)$ (first 3 non-zero terms) $x\ln(2 - x)$ (first 3 non-zero terms) $x\ln(4 - x^2)$ (first 2 non-zero terms) $\frac{x^2 + 6x - 4}{(x + 2)^2(x - 4)}$ (first 2 non-zero terms) $1 + \sin^2 x$ (first 3 non-zero terms) $\sqrt{(1 + x)(1 + x^2)}$ (first 4 terms)	
Know, or derive, the ***binomial series*** (which is a Maclaurin series):	

$(1 + x)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k \qquad (r \in \mathbb{R})$	
Use Maclaurin series to approximate values, for example: $\sin 0 \cdot 5$ $e^{0 \cdot 6}$ $\cos 0 \cdot 3$ $1 \cdot 4^{1/3}$ $\ln 1 \cdot 1$	
Know that a recurrence relation is sometimes known as an ***iterative scheme***	
Know that an ***iterative sequence*** is a sequence generated by an iterative scheme	
Know that each x_n is called an ***iterate***	
Calculate iterates given an iterative scheme	
Know that a ***fixed point*** (aka ***convergent*** or ***limit***) of an iterative scheme is a point a satisfying: $a = F(a)$	
Find fixed points of a recurrence relation, for example: $x_{n+1} = \frac{1}{2}\left(x_n + \frac{7}{x_n}\right)$ $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n^2}\right)$	
Use an iterative scheme of the form $x_n = F(x_n)$ to solve an equation $f(x) = 0$, where $x = F(x)$ is a rearrangement of the original equation	
Know that ***cobweb and staircase diagrams*** can be used to illustrate convergence (or divergence) of an iterative scheme	
Know that an iterative scheme has a fixed point if $\left\lvert \frac{dF}{dx} \right\rvert < 1$, where the derivative is evaluated at any point x in a small region of the fixed point	

Further Differential Equations

Skill	Achieved ?
Know that an ***n*** th ***order ordinary differential equation (ODE)*** is an equation containing (i) a function of a single variable and (ii) its derivatives up to the n^{th} derivative	
Know that a ***linear ODE*** is one of the form: $$\sum_{r=0}^{n} a_r(x)\,\frac{d^{(r)}y}{dx^{(r)}} = f(x)$$	
Know that an ***n*** th ***order linear ODE with constant coefficients*** is of the above form, but where all the a_r are constant	
Know that if $f(x) = 0$, the DE is called ***homogeneous***, whereas if $f(x) \neq 0$, the DE is called ***non-homogeneous*** (or ***inhomogeneous***)	
Know that a ***solution*** of a DE is a function that satisfies the DE	
Know that solutions of a DE are of 2 types: ***General solution*** ***Particular solutions***	
Know that the general solution of a DE has ***arbitrary constants***, whereas a particular solution has no arbitrary constants (as they are evaluated using ***initial conditions***)	
Know that the solutions of a homogeneous equation are called ***complementary functions (CF)***	
Know that a 1^{st} order linear ODE can be written in the form: $$\frac{dy}{dx} + P(x)\,y = f(x)$$	
Know that to solve a 1^{st} order linear ODE, the first step is to multiply the equation as written above by the ***integrating factor*** : $$e^{\int P(x)\,dx}$$	
Solve 1^{st} order linear ODEs using the integrating factor method, for example: $$\frac{dy}{dx} + \frac{y}{x} = x$$	

$x\dfrac{dy}{dx} - 3y = x^4$ $(x+1)\dfrac{dy}{dx} - 3y = (x+1)^4$	
Find a particular solution of a differential equation that is solved using the integrating factor method	
Know that a 2 nd order linear ODE with constant coefficients can be written in the form (inhomogeneous equation): $a\dfrac{d^2y}{dx^2} + b\dfrac{dy}{dx} + cy = f(x)$	
Know that the ***auxiliary equation*** (aka ***characteristic equation***) of the above DE is: $ap^2 + bp + c = 0 \quad (p \in \mathbb{C})$	
Know that the CF of the above DE can be written in one of 3 forms depending on the nature of the roots of the auxiliary equation	
Know that if the auxiliary equation has 2 real (distinct) roots p_1 and p_2, then the CF is of the form: $y_{CF} = Ae^{p_1 x} + Be^{p_2 x} \quad (A, B \in \mathbb{R})$	
Know that if the auxiliary equation has 1 real (repeated) root p, then the CF is of the form: $y_{CF} = (A + Bx)\,e^{px} \quad (A, B \in \mathbb{R})$	
Know that if the auxiliary equation has a pair of complex (conjugate) roots $p_1 = r + is$ and $p_2 = r - is$, then the CF is of the form: $y_{CF} = e^{rx}(A\cos sx + B\sin sx) \quad (A, B \in \mathbb{R})$	
Know that a ***particular integral (PI)*** is a solution of the inhomogeneous equation and is chosen to be of a similar form as the function $f(x)$: $f(x) = Cx + D \;\rightarrow\; y_{PI} = Sx + T$ $f(x) = Cx^2 + Dx + E \;\rightarrow\; y_{PI} = Sx^2 + Tx + U$ $f(x) = Ce^{px} \;\rightarrow\; y_{PI} = Se^{px}$	

 AH Checklist (Unit 3)

$f(x) = C\sin px \rightarrow y_{PI} = S\sin px + T\cos px$ $f(x) = C\cos px \rightarrow y_{PI} = S\sin px + T\cos px$	
Know slight variations of the last 3 PIs with an additional constant: $f(x) = Ce^{px} + D \rightarrow y_{PI} = Se^{px} + T$ $f(x) = C\sin px + D \rightarrow y_{PI} = S\sin px + T\cos px + U$ $f(x) = C\cos px + D \rightarrow y_{PI} = S\sin px + T\cos px + U$	
Know that if $f(x)$ is of the same form as a term in the CF, try the same form as $xf(x)$ for the PI; if this is of the same form as a term in the CF, try the same form as $x^2 f(x)$ for the PI	
Know that the general solution of the inhomogeneous equation is: $y_G = y_{CF} + y_{PI}$	
Find the general solution of 2 nd order linear ODEs with constant coefficients, for example: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1$ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\cos x$ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x$ $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20\sin x$ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$ $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$	

AH Checklist (Unit 3)

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12$ $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 24y = 3\sin x + 4\cos x + 12$ $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{2x}$	
Find the particular solution of a 2 nd order linear ODE with constant coefficients given initial conditions, such as: $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12$ given that $y = 2$ and $\frac{dy}{dx} = 1$ when $x = 0$.	

Number Theory and Further Proof Methods

Skill	Achieved ?
Know that when $A \Rightarrow B$, A is said to be ***sufficient*** for B and B is said to be ***necessary*** for A	
Know that in the biconditional $A \Leftrightarrow B$, A is said to be ***necessary and sufficient*** for B (and vice versa)	
Prove statements involving finite sums by induction, for example: $2 + 5 + 8 + \dots + (3n - 1) = \frac{1}{2}n(3n + 1) \quad (\forall n \in \mathbb{N})$ $\sum_{r=1}^{n} 3(r^2 - r) = (n - 1)n(n + 1) \quad (\forall n \in \mathbb{N})$ $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)} \quad (\forall n \in \mathbb{N})$ $\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{(n+1)} \quad (\forall n \in \mathbb{N})$	
Prove statements involving matrices using induction, for example: $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow A^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix} \quad (\forall n \in \mathbb{N})$ A, B square matrices with $AB = BA \Rightarrow A^n B = BA^n \quad (\forall n \in \mathbb{N})$	
Prove statements involving differentiation using induction, for example: $\frac{d^n}{dx^n}(x e^x) = (x + n)e^x \quad (\forall n \in \mathbb{N})$	
Prove other statements using induction, for example: $2^n > n^3 \quad (\forall n \geq 10)$ $5^n + 3$ is divisible by 4 $(\forall n \in \mathbb{N})$ $n! > n^2 \quad (\forall n > 3)$	

AH Checklist (Unit 3)

$8^n + 3^{n-2}$ is divisible by 5 $(\forall n \geq 2)$	
Know that the ***Division Algorithm*** states that, given $a, b \in \mathbb{N}$, $\exists!\ q$ (***quotient***), r (***remainder***) $\in \mathbb{N}$ satisfying: $a = bq + r \quad (0 \leq r < b)$	
Know that the ***greatest common divisor (GCD)*** (sometimes called the highest common factor) of 2 natural numbers is the biggest natural number that exactly divides those 2 numbers; the GCD of a and b is denoted GCD(a, b)	
Know that when $a = bq + r$, GCD(a, b) = GCD(b, r)	
Know that repeated application of the Division Algorithm until the GCD is reached is called the ***Euclidean Algorithm***	
Use the Euclidean Algorithm to find the GCD of 2 numbers	
Know that GCD(a, b) can be written as GCD(a, b) = $ax + by$	
Use the Euclidean Algorithm to write the GCD in the above form	
Know that numbers are ***coprime*** (aka ***relatively prime***) if their GCD = 1	
Use the Euclidean Algorithm to find integers x and y such that: $149x + 139y = 1$; $231x + 17y = 1$; $599x + 53y = 1$	
Know that a ***linear Diophantine equation*** is an equation of the form: $ax + by = c \quad (a, b, c, x, y \in \mathbb{Z})$	
Know that a linear Diophantine equation of the above form has a solution provided that GCD(a, b) divides c	
Know that if a linear Diophantine equation has a solution, then it has infinitely many solutions	
Solve linear Diophantine equations using the Euclidean Algorithm, for example: $4x + 3y = 8$; $5x + 7y = 14$; $15x + 27y = 21$	
Show that given linear Diophantine equations do not have any solutions, for example:	

$4x + 12y = 13$ $8x + 40y = 11$	
Know that any number may be written uniquely in base n as: $\sum_{i=0}^{k} r_{k-i} \; n^{k-i} \equiv (r_k \; r_{k-1} \; \ldots \; r_2 \; r_1 \; r_0)_n$	
Know that digits bigger than 9 are given by the abbreviations $A = 10$, $B = 11$, $C = 12$, $D = 13$, $E = 14$ etc.	
Use the Division Algorithm to convert a number in base 10 to another base by obtaining remainders	
Convert a number written in non-base 10 to base 10, for example: $(202)_3 = (20)_{10}$ $(4E)_{17} = (82)_{10}$	
Convert a number written in base 10 to non-base 10, for example: $(231)_{10} = (450)_7$ $(59)_{10} = (47)_{13}$	
Convert a number from one base to another, neither of which are base 10, by going through base 10 first,for example: $(1\,021)_4 = (73)_{10} = (81)_9$ $(1\,010\,011\,010)_2 = (666)_{10} = (556)_{11}$ $(5D)_{16} = (93)_{10} = (333)_5$ $(5A)_{12} = (70)_{10} = (55)_{13}$	