Higher Mathematics - Practice Examination B

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper I

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y \ + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \ . \ b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned} \sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\ \cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \\ \sin 2A &= 2\sin A \cos A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be answered

1. Differentiate $\frac{x^2+1}{\sqrt{x}}$, with respect to x ,

expressing your answer with positive indices. **(4)**

2. Two functions f and g are defined on the set of real numbers as follows :

$$f(x) = 2x-3 \quad , \quad g(x) = \frac{x+9}{4} \quad .$$

(a) Evaluate $f\left(g(-3)\right)$. **(1)**

(b) Find an expression , in its simplest form, for $g\left(f(x)\right)$. **(2)**

3. Given that $x=-1$ and $x=2$ are two roots of the equation $x^3+ax^2+2x+b=0$, establish the values of a and b and hence find the third root of the equation. **(5)**

4. A sequence is defined by the recurrence relation $U_{n+1} = 0\cdot 8\,U_n + 3$.

(a) Explain why this sequence has a limit as $n\to\infty$. **(1)**

(b) Find the limit of this sequence. **(2)**

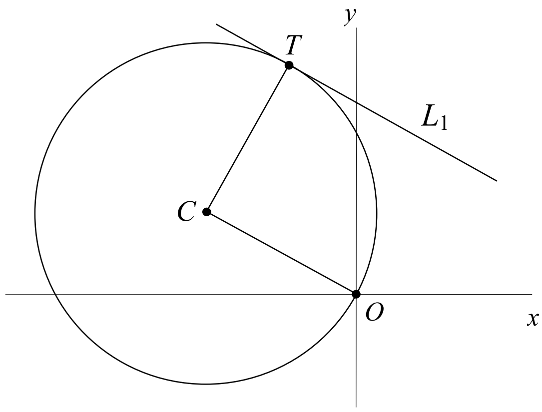
(c) Taking $U_0 = 10$ and L as the limit of the sequence, find n such that

$$L - U_n = 2\cdot 56$$ **(3)**

5. Find the equation of the line which passes through the point P(3,-5) and is parallel to the line passing through the points (-1,4) and (7,-2). **(4)**

6. The diagram below shows the circle with equation $x^2 + y^2 + 4x - 2y = 0$.

The line L_1 is a tangent to the circle at the point T and has as its equation $x+2y=5$.

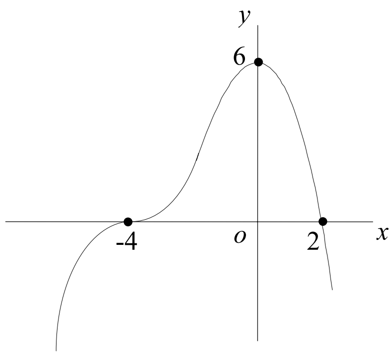


(a) Find the coordinates of point T. **(3)**

(b) Given that the centre of the circle is $C(-2,1)$, show that angle TCO is a right angle. **(3)**

7. The graph of $y = f(x)$ is shown below.

Sketch the graph of $y = f'(x)$. **(3)**



8. Two circles, which do not touch or overlap, have as their equations

$$(x-15)^2 + (y-6)^2 = 40 \quad \text{and} \quad x^2 + y^2 - 6x - 4y + 3 = 0 \ .$$

(a) Show that the **exact** distance between the centres of the two circles is $4\sqrt{10}$ units. **(3)**

(b) Hence show that the shortest distance between the two circles is equal to the radius of the smaller circle. **(4)**

9. Evaluate $\int_0^2 (4-3x)^2 \, dx$ **(4)**

10. The picture below shows a small section of a larger circuit board.

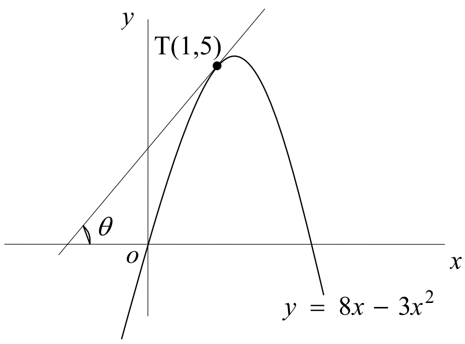
Relative to rectangular axes the points P , Q and R have as their coordinates (-8,3,1) , (-2,-6,4) and (2,-12,6) respectively.
Prove that the points P , Q and R are collinear, and find the ratio $PQ : QR$. **(4)**

11. A function is defined as $f(x) = \dfrac{3p}{x^2 - 18x + 87}$, for $x \in R$ and p is a constant.

(a) Express the function in the form $f(x) = \dfrac{3p}{(x-a)^2 + b}$, and hence state the maximum value of f in terms of p . **(4)**

(b) Given now that $p = \dfrac{2}{\sqrt{2} + 1}$ show that the **exact** maximum value of f is $\sqrt{2} - 1$. **(2)**

12. The diagram below shows the parabola with equation $y = 8x - 3x^2$ and the line which is a tangent to the curve at the point T(1,5).



Find the size of the angle marked θ , to the nearest degree. **(4)**

13. (a) Points E , F and G have coordinates (-1,2,1) , (1,3,0) and (-2,-2,2) respectively.

Given that $3\overrightarrow{EF} = \overrightarrow{GH}$, find the coordinates of the point H. **(3)**

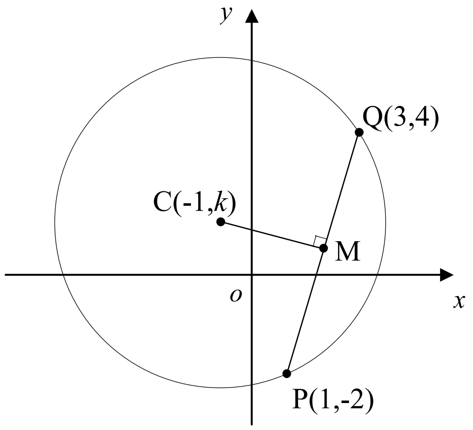
(b) Hence calculate $\left|\overrightarrow{EH}\right|$. **(2)**

14. Solve algebraically the equation

$$3\cos 2x^\circ - 9\cos x^\circ = 12 \qquad \text{for} \qquad 0 \le x < 360.$$ **(6)**

15. A circle has as its centre C(-1,k) .

A chord PQ is drawn with end - points P(1,-2) and Q(3,4) as shown in the diagram.



(a) Establish the gradient of CM. **(2)**

(b) Write down the coordinates of M. **(1)**

(c) Find the value of k . **(3)**

16. Given that $g(x) = \left(\frac{1}{2}x^2 - 1\right)^4$, find the value of $g'(2)$. **(4)**

17. (a) Given that $2\log_x y = \log_x 2y + 2$ find a relationship connecting x and y . **(4)**

(b) Hence find y when $x = \frac{1}{4}y$ and $y > 0$. **(2)**

[END OF QUESTION PAPER]

Higher Mathematics Practice Exam B   Marking Scheme - Paper 1

1. For $x^{-\frac{1}{2}}(x^2+1)$ .... (1)

For $x^{\frac{3}{2}} + x^{-\frac{1}{2}}$ .... (1)

For $\frac{d}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$ .... (1)

Then ans. $= \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2x^{\frac{3}{2}}}$ .... (1) **[4 marks]**

2. (a) For answer $f\left(g(-3)\right) = 0$ .... (1) **[1 mark]**

(b) For $g\left(f(x)\right) = \frac{(2x-3)+9}{4}$ .... (1)

for ans. $g\left(f(x)\right) = \frac{1}{2}(x+3)$ (or equiv.) .... (1) **[2 marks]**

3. For realising to use synthetic division .... (1)

For setting up synth. div. tables correctly .... (1)

For $a+b=3$ and $4a+b=-12$ .... (1)

For solving system to ans. $a=-5$ and $b=8$ .... (1)

For finding third root i.e. $x = 4$ .... (1) **[5 marks]**

4. (a) Explanation i.e. $-1 < a < 1$ etc. (or just words) .... (1) **[1 mark]**

(b) For knowing $L = \frac{b}{1-a}$ (or equiv.) .... (1)

For answer $L = 15$ .... (1) **[2 marks]**

(c) For $U_1 = 0\cdot8(10) + 3 = 11$ .... (1)

then to $U_3 = 12\cdot44 \quad \therefore\ 15 - 12\cdot44 = 2\cdot56$ .... (1)

for stating the requested ans. i.e. $n = 3$ .... (1) **[3 marks]**

5. For $m_L = -\frac{3}{4}$ .... (1)

For $y+5 = -\frac{3}{4}(x-3)$ .... (2)

i.e. correct point with correct gradient

For answer $3x+4y+11=0$ (or equiv.) .... (1) **[4 marks]**

6. (a) For solving a system i.e. $\left.\begin{array}{l} x+2y=5 \\ x^2+y^2+4x-2y=0 \end{array}\right\}$ .... (1)

For first coord. i.e. $y = 3$ .... (1)

For complete point T(-1,3) .... (1) **[3 marks]**

(b) For strategy e.g. using grad. or pyth etc. .... (1)

For seeing through to i.e. $m_{OC} \times m_{OT} = -\frac{1}{2} \times 2$ .... (1)

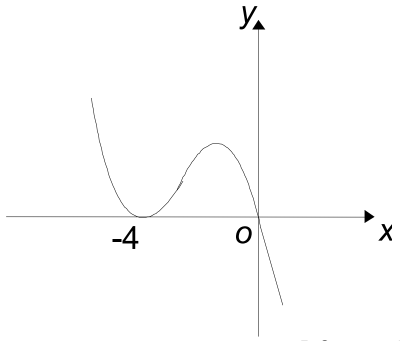
then ans. $m_{OC} \times m_{OT} = -\frac{1}{2} \times 2 = -1 \therefore$ *right angle* .... (1)

(pupils may simply equate gradients i.e. L_1 and OC .... full marks) **[3 marks]**

7. For correct shape .... (1)

For correct roots .... (1)

For annotation .... (1)



[3 marks]

8. (a) For finding centres, one mark each - (15,6) .... (1)

(3,2) .... (1)

For Pyth. or dis. form. to answer - $4\sqrt{10}$ .... (1) **[3 marks]**

(b) For each radius one mark - $r_1 = \sqrt{40} = 2\sqrt{10}$ .... (1)

from $r = \sqrt{f^2 + g^2 - c}$ - $r_2 = \sqrt{10}$ .... (1)

For $\sqrt{10} + 2\sqrt{10} + \text{gap} = 4\sqrt{10}$ (or equiv.) .... (1)

$\therefore$ short. d = $\sqrt{10}$ = the radius of the smaller circle .... (1)

(*note : no marks off for approximation throughout*) **[4 marks]**

9. For $\displaystyle\int_0^2 (16 - 24x + 9x^2)\, dx$ .... (1)

$\left[\, 16x - 12x^2 + 3x^3 \right]_0^2$ .... (1)

$(32 - 48 + 24) - (0)$ .... (1)

For answer 8 .... (1) **[4 marks]**

10. For selecting correct displacements i.e $\overrightarrow{PQ}$ *and* $\overrightarrow{QR}$ .... (1)

For both correct $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -9 \\ 3 \end{pmatrix}$ *and* $\overrightarrow{QR} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ .... (1)

For collinear statement i.e. since $\overrightarrow{PQ} = \frac{3}{2}\overrightarrow{QR}$

then P , Q and R are collinear (or equiv.) .... (1)

For correct ratio $PQ : QR = 3 : 2$ .... (1) **[4 marks]**

11. (a) For $\left[(x-9)^2 - 81 \right] + 87$ .... (1)

for $f(x) = \dfrac{3p}{(x-9)^2 + 6}$ .... (1)

then stated or implied $(x-9)^2 + 6 \rightarrow \min = 6$ .... (1)

$\therefore \ f_{\max} = \dfrac{3p}{6} = \frac{1}{2}p$ .... (1) **[4 marks]**

(b) For $f_{\max} = \dfrac{1}{2} \cdot \dfrac{2}{\sqrt{2}+1} = \dfrac{1}{\sqrt{2}+1}$ .... (1)

then $f_{\max} = \dfrac{1}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} = \dfrac{\sqrt{2}-1}{1} = ans.$ .... (1) **[2 marks]**

12. For $\dfrac{dy}{dx} = 8 - 6x$ .... (1)

For $m_{\tan} = 8 - 6(1) = 2$ .... (1)

For $\tan\theta = m = 2$ (stated or implied) .... (1)

For ans. $\therefore \ \theta = 63^\circ$ .... (1) **[4 marks]**

(*note : no marks off for not rounding*)

13. (a) For $\overrightarrow{EF} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ .... (1)

For $\begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix} = \underset{\sim}{h} - \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$ .... (1)

then for ans. $\underset{\sim}{h} = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = H(4,1,-1)$ .... (1) **[3 marks]**

(b) For $\underset{\sim}{h} - \underset{\sim}{e} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$ .... (1)

then for ans. mag. $= \sqrt{30}$ .... (1) **[2 marks]**

14. For re-arranging to zero and removing the common factor (whenever) .... (1)
For using correct replacement $2\cos^2 x^\circ - 1$ .... (1)
For line - $3(2\cos^2 x^\circ - 3\cos x^\circ - 5) = 0$ .... (1)
then for factorising and solving to $\cos x^\circ = \frac{5}{2}$ *or* $\cos x^\circ = -1$ .... (1)
For rejecting $\cos x^\circ = \frac{5}{2}$ as having no solution .... (1)
For final answer - $x = 180^\circ$ .... (1) **[6 marks]**

15. (a) For gradient of PQ $m_{PQ} = 3$ .... (1)
For gradient of perpendicular $m = -\frac{1}{3}$ .... (1)

(b) For coordinates of mid-point (2,1) .... (1)

(c) For selecting a strategy .... (1)

For equat. gradients .... (1)
For $k = 2$ .... (1)

or For finding and using the equation of the perpendicular $3y + x = 5$ .... (2)

[6 marks]

16. For $g'(x) = 4(\frac{1}{2}x^2 - 1)^3 \,.\, x$ .... (2)
(split as follows $4(\frac{1}{2}x^2 - 1)^3$; $\times x$ ..... 1 *mark each*)
For $g'(2) = 4(2)\left(\frac{1}{2}(2^2) - 1\right)$ .... (1)
For answer $g'(2) = 8$ .... (1) **[4 marks]**

17. (a) For $2\log_x y - \log_x 2y = 2$ .... (1)
then $\log_x y^2 - \log_2 2y = 2$ .... (1)
then $\log_x \dfrac{y^2}{2y} = 2 \Rightarrow \log_x \frac{1}{2}y = 2$ .... (1)
For $x^2 = \frac{1}{2}y \Rightarrow y = 2x^2$ (or equiv.) .... (1) **[4 marks]**

(b) For $y = 2(\frac{1}{4}y)^2$ (or equiv.) .... (1)
then for $y = \frac{1}{8}y^2 \Rightarrow$ ans. $y = 8$ .... (1) **[2 marks]**

Total	**83 marks**

Higher Mathematics - Practice Examination B

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MATHEMATICS

Higher Grade - Paper II

Time allowed - 2 hours 30 minutes

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \\ \sin 2A &= 2\sin A \cos A \end{aligned}$$

Table of standard derivatives:

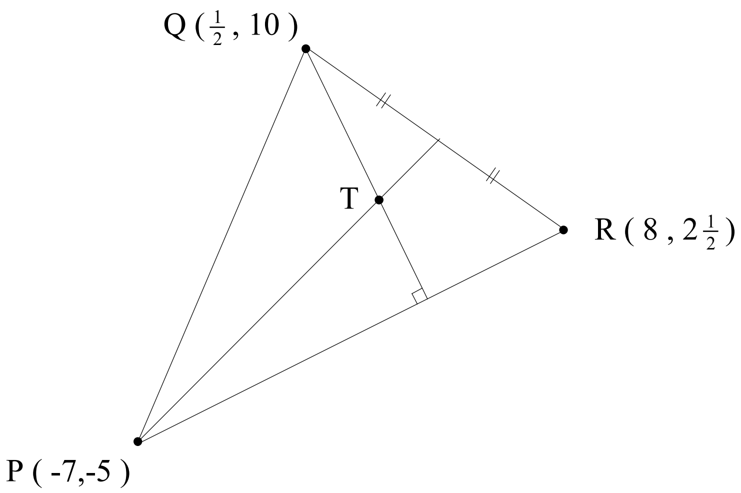
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be answered

1. In the diagram below triangle PQR has vertices as shown.



(a) Find the equation of the median from P to QR . **(3)**

(b) Find the equation of the altitude from Q to PR . **(4)**

(c) Hence find the coordinates of the point T where these two lines cross. **(3)**

2. A circle, centre C, has as its equation $x^2 + y^2 - 4x - 2y - 15 = 0$.

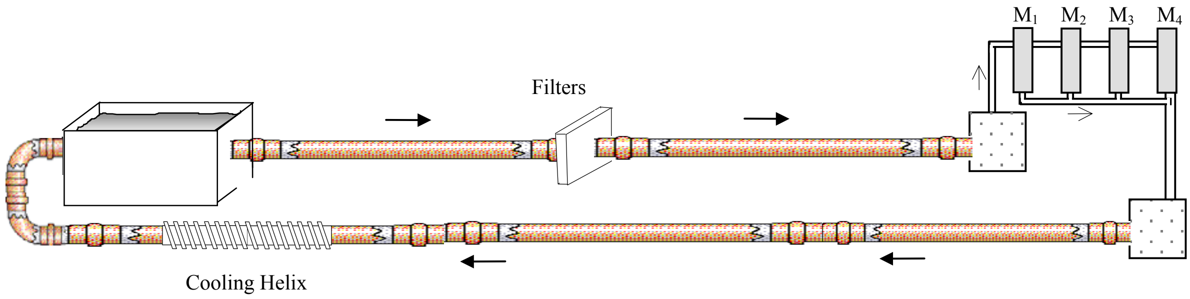
(a) Show that the line with equation $y + 2x = 15$ is a tangent to this circle and state the coordinates of T , the point of tangency. **(5)**

(b) The point P(k , -5) lies on this line of tangency. Find k . **(1)**

(c) Establish the equation of the circle which passes through the points C , T and P. **(4)**

3. Industrial coolant is a water/oil based liquid used to cool down metal components during their manufacture. It is continuously poured over the component and the cutting tool.

The diagram below (which is not to scale) shows how the coolant is pumped from a main holding tank to each machine in the factory and is then recycled back to the tank.



The coolant has one other important and expensive ingredient, namely an anti-bacterial agent. This agent, while active, hinders the growth of bacteria which thrive in the heated coolant. If this bacteria is allowed to multiply it poses a real health risk for the machine operators.

A company works the following system : At the beginning of each week (Monday morning) the liquid is drained from the system and replaced by new coolant, 40 units of the anti-bacterial agent is immediately added via the filter system.

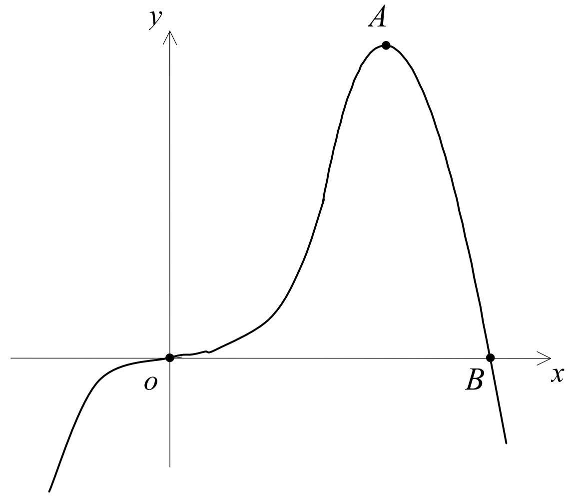
It is known that the anti-bacterial agent decays at the rate of 13% per working day and that this decayed amount is now said to be non-active.

(a) How many units of the anti-bacterial agent are still active at the end of a normal working week (Friday evening)?
Give your answer to the nearest hundredth of a unit. **(3)**

(b) New government guidelines have just been issued which state that the minimum amount of active anti-bacterial agent which should be present in industrial coolant is 28 units.
The company decides to meet these new guidelines by topping up the agent at the end of each working day.
Calculate the **minimum** number of **whole units** which should be added at the end of each day in order to keep the active agent above the 28 units until the end of the working week. **(5)**

(c) The company had of course looked at the alternative of simply adding more than the 40 units of the anti-bacterial agent at the beginning of the week.
Consider this option and comment. **(3)**

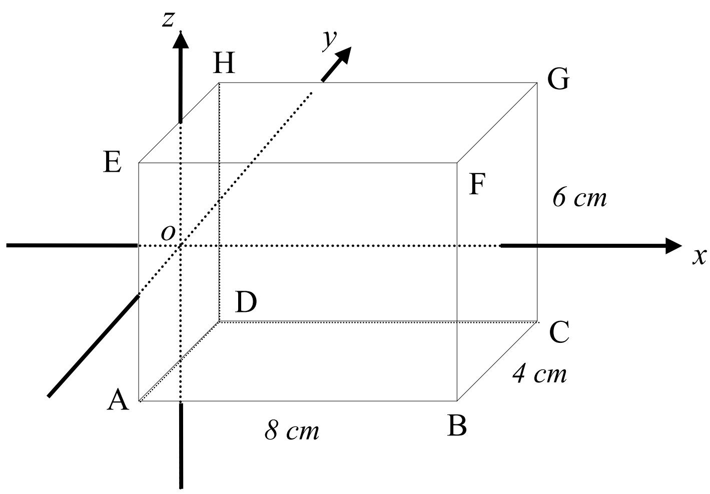
4. The curve shown below has as its equation $y = 8x^3 - 2x^4$.



(a) Find the coordinates of the points A and B . **(7)**

(b) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}$. **(5)**

5. A cuboid is placed relative to a set of coordinate axes as shown in the diagram.
The cuboid has dimensions *8 cm* by *4 cm* by *6 cm*.



The origin of the axes is at the intersection point of the diagonals ED and HA and *1 unit* represents *1 cm* .

(a) Find the coordinates of B , D and G. **(3)**

(b) Hence calculate the size of angle BDG. **(6)**

6. The main power source used to run the looms at the New Lanark cotton mills was water. Each mill had its own large water wheel which consisted of a central hub and an array of trough like blades. The wheels were driven by water being poured over the wheel thus filling the troughs and allowing the weight of the water to turn the wheel in a continuous process.
The diagram below shows the structure of one of these trough-blades.

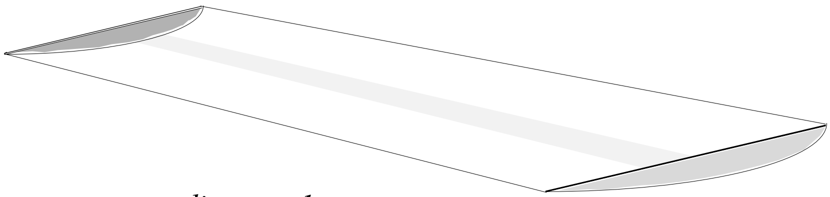


diagram 1

The shaded end-section consists of part of a curve and a straight line.
This end-section can be rotated and placed on a set of rectangular axes as shown in *diagram 2*.

The axes are not drawn to scale.

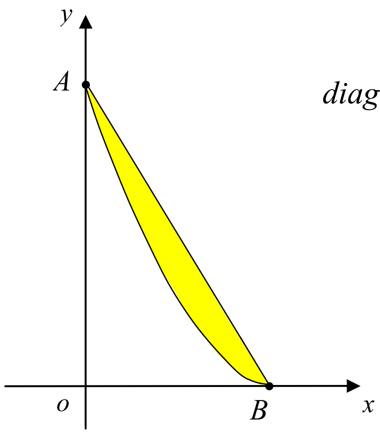


diagram 2

The curved section from A to B is part of the curve $y = x^3 - 2x^2 - 4x + 8$.

(a) State the coordinates of point A. **(1)**

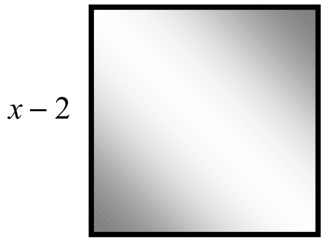
(b) The line AB is a tangent to the curve at A.
Hence, or otherwise, show that this line has as its equation $y = 8 - 4x$. **(3)**

(c) Calculate, in **square units**, the shaded area in *diagram 2*. **(5)**

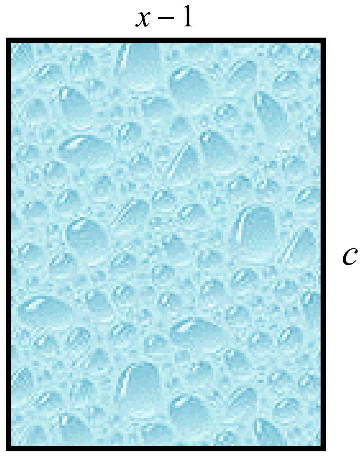
(d) Given that the scale in *diagram 2* is 1 unit = 10cm, write down the area of the end-section in **square centimetres**. **(1)**

(e) Hence calculate the volume of water this trough can hold when full, given that the trough has a length of 6 metres. **Express your answer in litres**. **(2)**

7. The two diagrams shown are design logo backgrounds.
Design 1 is a square of side $x-2$
Design 2 is a rectangle measuring $x-1$ by c .
All lengths are in centimetres.



Design 1



Design 2

(a) The area of *Design 2* is $24\,cm^2$ **more** than that of *Design 1*.
By equating the areas show that the following equation can be constructed

$$x^2 - (c+4)x + (c+28) = 0$$ **(3)**

(b) Hence find the value of c if the equation $x^2 - (c+4)x + (c+28) = 0$ has equal roots. **(5)**

(c) Using this value for c , solve the equation for x and hence calculate the area of each design. **(3)**

8. The luminosity, L units, emitting from a pulsing light source is given by the formula

$$L = \cos 36t^\circ + \sqrt{3}\sin 36t^\circ + 2 \ ,$$

where t is the time in seconds from switch on.

(a) Express L in the form $R\cos(36t-\alpha)^\circ + 2$, where $R > 0$ and $0 \le \alpha \le 360$. **(4)**

(b) Sketch the graph of L for $0 \le t \le 10$, indicating all relevant points. **(5)**

(c) If $L = 2\cdot 5$ the light source has the same luminosity as the surrounding light.
When will this **first** occur during this ten second period ?
Give your answer correct to the nearest tenth of a second. **(3)**

[END OF QUESTION PAPER]

Higher Mathematics Practice Exam B Marking Scheme - Paper 2

1.	(a)	For mid-pt. of QR - $M(4\frac{1}{4}, 6\frac{1}{4})$	....	(1)	
		For $m_{PM} = 1$	....	(1)	
		For equation of median $y = x + 2$	....	(1)	**[3 marks]**
	(b)	For $m_{PR} = \frac{1}{2}$	....	(1)	
		For $m_{alt.} = -2$	....	(1)	
		For corr. pt. & grad. to $y - 10 = -2(x - \frac{1}{2})$	....	(1)	
		For answer $y + 2x = 11$ (or equiv.)	....	(1)	**[4 marks]**
	(c)	For solving a system i.e. $\left.\begin{matrix} y = x + 2 \\ y + 2x = 11 \end{matrix}\right\}$	....	(1)	
		For first coordinate x or y , eg. $x = 3$	....	(1)	
		For complete point T(3,5)	....	(1)	**[3 marks]**
2.	(a)	For knowing to solve as a system	....	(1)	
		For $x^2 + (15 - 2x)^2 - 4x - 2(15 - 2x) - 15 = 0$	....	(1)	
		then for $5(x - 6)(x - 6) = 0 \quad \therefore \quad x = 6$	....	(1)	
		For finding y - coordinate - y = 3 then T(6,3)	....	(1)	
		For stating that one point represents a tangent or equivalent (i.e. $b^2 - 4ac = 0, etc.$)	....	(1)	**[5 marks]**
	(b)	For answer k = 10	....	(1)	**[1 mark]**
	(c)	For realising CP is a diameter	....	(1)	
		For establishing centre as (6,-2)	....	(1)	
		For calculating r *or* r^2 i.e. $r^2 = 25$	....	(1)	
		then for ans. $(x - 6)^2 + (y + 2)^2 = 25$	....	(1)	**[4 marks]**

(*note: other methods possible i.e. system 3 unknowns, intersection of the perpendicular bisectors*)

3.	(a)	For correct multiplier $a = 0 \cdot 87$	....	(1)	
		For calc. $U_5 = (0 \cdot 87)^5 \times 40$ (or equiv.)	....	(1)	
		For ans $= 19 \cdot 94$ units	....	(1)	**[3 marks]**
	(b)	For setting up $U_{n+1} = (0 \cdot 87)40 + 1$ *or* 2 (or equiv.)	....	(1)	
		For looking at different values of b	....	(1)	
		For setting out calculations to answers	....	(1)	
		For considering the ***lower*** value **before the addition**	....	(1)	
		For discovering that +2 doesn't work $\therefore$ $b = 3$ *units*	....	(1)	**[5 marks]**
	(c)	For realising total for (b) is 40 + 12 = 52 *units*	....	(1)	
		For calculations as (a) for different U_0 until $U_0 = 57$	....	(1)	
		For conclusion i.e. (b) is better saves 5 units/week	....	(1)	**[3 marks]**

4.	(a)	<u>For B</u>	For $8x^3 - 2x^4 = 0$	....	(1)	
			For $x = 0$ or $x = 4$	....	(1)	
			For giving B(4,0)	....	(1)	
		<u>For A</u>	For $dy/dx = 24x^2 - 8x^3$	....	(1)	
			then $24x^2 - 8x^3 = 0$	....	(1)	
			For solv. to $x = 0$ or $x = 3$	....	(1)	
			For using $x = 3$ and finding y $\therefore$ A(3,54)	....	(1)	**[7 marks]**

(b) For using derivative to find m of tangent. .... (1)

For sub. $x = \frac{1}{2}$ into dy/dx giving $m_{\tan} = 5$ .... (1)

For sub. $x = \frac{1}{2}$ into " y = " for point $(\frac{1}{2}, \frac{7}{8})$ .... (1)

For using y - b = m(x - a) (or equiv.) with point and gradient .... (1)

For answer from above $8y = 40x - 13$ (or equiv.) .... (1) **[5 marks]**

5. (a) For B(8,-2,-3) , D(0,2,-3) and G(8,2,3) (1 each) **[3 marks]**

(b) For selecting suitable vectors i.e. $\vec{DB}$ *and* $\vec{DG}$ .... (1)

For $\vec{DB} = \begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix}$ *and* $\vec{DG} = \begin{pmatrix} 8 \\ 0 \\ 6 \end{pmatrix}$ .... (1)

For scalar product $\vec{DB} \cdot \vec{DG} = 64$ .... (1)

For both magnitudes $4\sqrt{5}$ *and* 10 .... (1)

For $\cos\theta = \dfrac{64}{40\sqrt{5}}$ (or equiv.) .... (1)

For ans. angle BDG = $44 \cdot 3^\circ$ .... (1) **[6 marks]**

6. (a) For ans. A(0,8) .... (1) **[1 mark]**

(b) For $m = \dfrac{dy}{dx} = 3x^2 - 4x - 4$ .... (1)

for @ $x = 0$, $m = -4$ .... (1)

for using (0,8) and m = -4 to ans. .... (1) **[3 marks]**

(pupils may use poly. to est. point B etc. - assign own marks)

(c) For $Area = \int_0^2 \left((8 - 4x) - (x^3 - 2x^2 - 4x + 8)\right) dx$ .... (2)

(*or two separate integrals*) (1 for limits + 1 for setting up Int.)

for $Area = \left[\dfrac{2x^3}{3} - \dfrac{x^4}{4}\right]_0^2$ .... (1)

for $Area = (16/3 - 4) - (0)$ .... (1)

then ans. $Area = 1\frac{1}{3}\ units^2$ .... (1) **[5 marks]**

(d) For $1\frac{1}{3} \times 100 = 133\frac{1}{3}\ cm^2$ ($133 \cdot 3$ o.k.) .... (1) **[1 marks]**

(e) For $V = 133\frac{1}{3} \times 600$ .... (1)

for ans. $V = 80000\ cm^3 = 80\ litres$ .... (1) **[2 marks]**

7. (a) For $(x-2)^2 = c(x-1) - 24$ .... (1)

then $x^2 - 4x + 4 - cx + c + 24 = 0$ .... (1)

for $x^2 - (c+4)x + (c+28) = 0$ .... (1) **[3 marks]**

(b) For stating "for equal roots - $b^2 - 4ac = 0$" .... (1)

For $a = 1$, $b = -(c+4)$ and $c = c + 28$ .... (1)

For sub. then to $c^2 + 4c - 96 = 0$ .... (1)

For solving to $c = 8$ or $c = -12$ .... (1)

For discarding -12 (-ve length) $\therefore$ $c = 8$ .... (1) **[5 marks]**

(c) For $x^2 - (8+4)x + (8+28) = 0$ .... (1)

The solved to $x = 6\ cm$ .... (1)

For areas $D_1 = 16\ cm^2$, $D_2 = 40\ cm^2$ .... (1) **[3 marks]**

8. (a) For $R = 2$ .... (1)

For $\tan\alpha = \sqrt{3}/1$ .... (1)

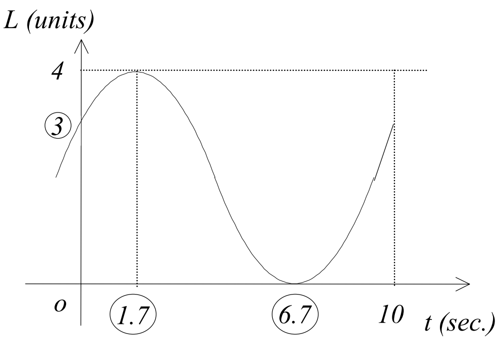
For realising 1st Quad. then $\alpha = 60^\circ$ .... (1)

For $L = 2\cos(36t - 60)^\circ + 2$ .... (1) **[4 marks]**

(b) For $L_{\max} = 4$ and $L_{\min} = 0$ .... (1)

For drawing of graph .... (1)

For each number circled 1 mark .... (3) **[5 marks]**



(c) For $2\cos(36t - 60)^\circ + 2 = 2\cdot5$ .... (1)

$36t - 60 = 75\cdot5$ .... (1)

For ans $t = 3\cdot8$ seconds .... (1)

[3 marks]

Total 87 marks