Higher Mathematics - Practice Examination E

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper I

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$
$$\sin 2A = 2 \sin A \cos A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

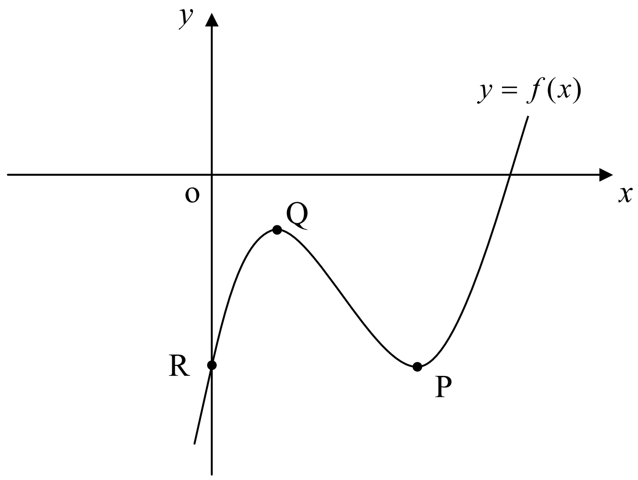
1. Find $f'(x)$ when $f(x) = \dfrac{x^2 - \sqrt{x}}{x}$. **(4)**

2. A sequence of numbers is defined by the recurrence relation $U_{n+1} = pU_n + q$, where p and q are constants.

(a) Given that $U_0 = 3$, $U_1 = 2$ and $U_2 = -2$, find **algebraically** , the values of p and q . **(3)**

(b) Hence find U_3 . **(1)**

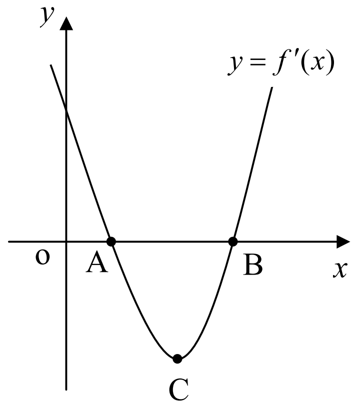
3. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x - 5$ is shown below. The graph has two turning points at P and Q and a y-intercept point at R.



(a) Find the equation of the tangent to the curve at R. **(4)**

(b) The sketch of the derived function $y = f'(x)$ from the above graph, is shown opposite.

Find the coordinates of A, B and C. **(4)**



4. Given that $x - 2$ is a factor of $x^3 + x^2 - (k+1)x - 4$, find the value of k and hence fully factorise the expression when k takes this value. **(4)**

5. The power , P , emitting from a wave generator is given by the formula

$$P = 2\sin 15t^\circ + \sqrt{5}\cos 15t^\circ + 7 ,$$

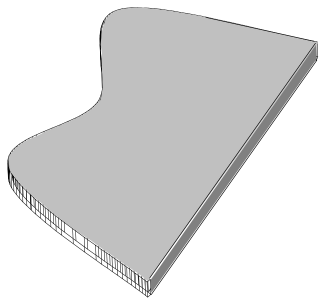
where t is the time elapsed, in seconds, from switch on.

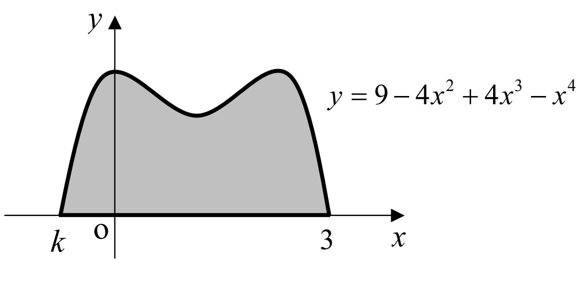
(a) Express P in the form $k\cos(15t - \alpha)^\circ + 7$, where $k > 0$ and $0 \le \alpha \le 360$. **(4)**

(b) Hence find t when $P = 9$, where t lies in the interval $0 < t < 12$. **(4)**

6. Show that $\int_0^{\frac{\pi}{12}} (1 + \cos 2x)\, dx = \frac{1}{12}(\pi + 3)$. **(4)**

7. A modern table top is an abstract bow shape.
The table top can be modelled by a straight line and a curve.
When rotated and placed on a set of rectangular axes the straight edge can be placed along the x-axis and the curve given the equation $y = 9 - 4x^2 + 4x^3 - x^4$, as shown below.

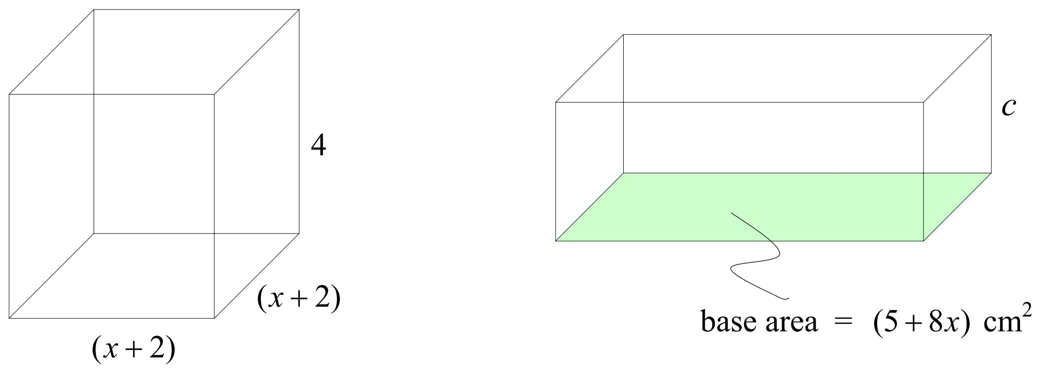




(a) Given that one root of the equation $9 - 4x^2 + 4x^3 - x^4 = 0$ is 3, find the value of the other root k. **(3)**

(b) Calculate the area of the table top in square units. **(4)**

8. The two cuboids below have equal volumes.
All lengths are in centimetres.

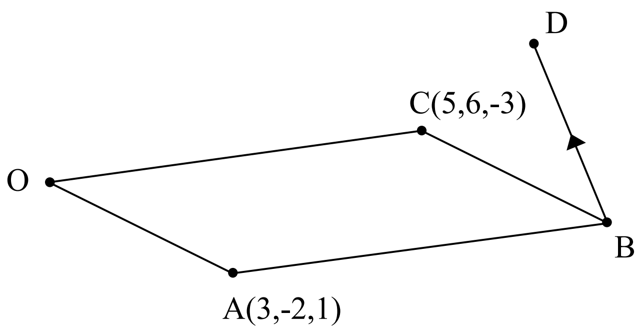


(a) By equating the two volumes show that the following equation can be constructed ....

$$4x^2 + (16 - 8c)x + (16 - 5c) = 0$$ **(2)**

(b) Given that $c > 0$, find the value of c for which the equation $4x^2 + (16 - 8c)x + (16 - 5c) = 0$ has **equal** roots. **(4)**

9. The parallelogram OABC, where O is the origin, has two more of its vertices at A(3,-2,1) and C(5,6,-3), as shown in the diagram.



(a) Find $\overrightarrow{AC}$ in component form. **(1)**

(b) D is a point such that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix}$.

Show that A, C and D are collinear. **(4)**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**ans:** $f'(x) = 1 + \frac{1}{2}x^{-3/2}$ **4 marks** •1 for dealing with the denominator •2 correct preparation •3 for diff. 1st term •4 for diff. 2nd term	•1 $f(x) = x^{-1}(x^2 - \sqrt{x})$ •2 $= x - x^{-\frac{1}{2}}$ •3 $f'(x) = 1 + ...$ •4 $= ...... + \frac{1}{2}x^{-3/2}$
2.	**(a) ans:** $p = 4 \quad , \quad q = -10$ **3 marks** •1 setting up a system •2 finding p •3 finding q **(b) ans:** -18 **1 mark** •1 sub. to answer	(a) •1 $2 = 3p + q$, $-2 = 2p + q$ (or equiv.) •2 $p = 4$ •3 $q = -10$ (b) •1 $U_3 = 4(-2) - 10 = -18$
3.	**(a) ans:** $y = 9x - 5$ **4 marks** •1 establishing coords. of R •2 strategy of differentiation for m •3 finding m •4 finding equation of tangent **(b) ans:** A(1,0) , B(3,0) , C(2,-3) **4 marks** •1 for knowing to relate S.P's to x-axis •2 solving derivative to zero •3 for A and B (x = 1 or x = 3 is o.k.) •4 for C	(a) •1 R(0,-5) •2 $f'(x) = 3x^2 - 12x + 9 = m$ (stated or implied) •3 $f'(0) = 9 = m$ •4 $y = 9x - 5$ (b) •1 attempting to find S.P.'s for roots •2 $3x^2 - 12x + 9 = 0$ •3 $x = 1$ *or* $x = 3$ ∴ A(1,0) , B(3,0) •4 sub. 2 in deriv. ⇒ C(2,-3)
4.	**ans:** $k = 3$, $(x-2)(x+2)(x+1)$ **4 marks** •1 setting up synthetic div. •2 finding k •3 sub. k and establishing quotient •4 finding remaining factors	•1 2 1 1 -(k+1) -4 •2 $2(5 - k) = 4 \Rightarrow k = 3$ •3 *quo.* ⇒ 1 3 2 $\Rightarrow x^2 + 3x + 2$ •4 ...... $(x + 2)(x + 1)$
5.	**(a) ans:** $P = 3\cos(15t - 41 \cdot 8)^\circ + 7$ **4 marks** •1 finding k •2 knowing how to find angle •3 knowing 1st quadrant … correct angle •4 for writing form **(b) ans:** $t \approx 6$ sec*onds* **4 marks** •1 knowing to solve to 9 •2 dividing by 3 •3 solving to angle •4 answer	(a) •1 $k = 3$ •2 $\tan\alpha = 2/\sqrt{5}$ •3 $\alpha = 41 \cdot 8^\circ$ •4 $P = 3\cos(15t - 41 \cdot 8)^\circ + 7$ (b) •1 $3\cos(15t - 41 \cdot 8)^\circ + 7 = 9$ •2 $\cos(15t - 41 \cdot 8)^\circ = \frac{2}{3}$ •3 $(15t - 41 \cdot 8)^\circ = 48 \cdot 2^\circ$ •4 $t \approx 6$ sec*onds*

	Give 1 mark for each •	Illustration(s) for awarding each mark
6.	**ans: proof** **4 marks** •1 For integrating •2 For substitution •3 For simplifying •4 For common factor to ans	•1 $\left[x+\frac{1}{2}\sin 2x\right]_0^{\pi/12}$ •2 $(\frac{\pi}{12}+\frac{1}{2}\sin\frac{\pi}{6}) - (0+\frac{1}{2}\sin 0)$ •3 $\frac{\pi}{12}+\frac{1}{4}$ •4 $\frac{1}{12}(\pi+3)$
7.	**(a) ans:** $k=-1$ **3 marks** •1 attempting to set up synth. div. •2 for finding quotient •3 for finding factor of quotient **(b) ans:** $29\frac{13}{15}$ units^2 **4 marks** •1 setting up correct integral •2 integrating •3 attempting to substitute correctly •4 calculating the correct answer	(a) •1 3 | -1 4 -4 0 9 ... 0 •2 -1 1 -1 -3 •3 -1 | -1 1 -1 -3 / 1 -2 3 / -1 2 -3 0 $\therefore k=-1$ (b) •1 $\int_{-1}^{3} 9-4x^2+4x^3-x^4\ dx$ •2 $\left[9x-\frac{4x^3}{3}+x^4-\frac{x^5}{5}\right]_{-1}^{3}$ •3 (..........) − (...........) •4 $(23\frac{2}{5})-(-6\frac{7}{15})=29\frac{13}{15}$
8.	**(a) ans:** proof **2 marks** •1 equating volumes •2 expanding and rearranging as required **(b) ans:** $c=\frac{11}{4}$ cm **4 marks** •1 understanding procedure •2 for selecting and sub. a, b and c •3 expanding and arranging •4 factorising to answer	(a) •1 $4(x+2)^2=c(5+8x)$ •2 $4x^2+16x-8cx+16-5c=0$ to ans. (b) •1 $b^2-4ac=0$ (stated or implied) •2 $a=4,\ b=16-8c,\ c=16-5c$ •3 $64c^2-176c=0$ •4 $16c(4c-11)=0 \Rightarrow c=\frac{11}{4}$, $(c\neq 0)$
9.	**(a) ans:** $\vec{AC}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$ **1 mark** •1 for answer **(b) ans:** proof **4 marks** * *there are a few different methods for (b)* •1 for establishing coords. of B •2 for establishing coords. of D •3 for $\vec{CD}$ in component form •4 ans.(common point should be mentioned)	(a) •1 $\underset{\sim}{c}-\underset{\sim}{a}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$ (b) •1 B(8,4,-2) •2 D(6,10,-5) •3 $\vec{CD}=c-d=\begin{pmatrix}1\\4\\-2\end{pmatrix}$ •4 since $\vec{AC}=2\vec{CD}$,and C is a common point, then A, C and D are collinear

Total 50 marks

Higher Mathematics - Practice Examination E

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper II

Time allowed - 2 hours 40 mins

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\,,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin\left(A \pm B\right) = \sin A \cos B \pm \cos A \sin B$$
$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A \cos A$$

Table of standard derivatives:

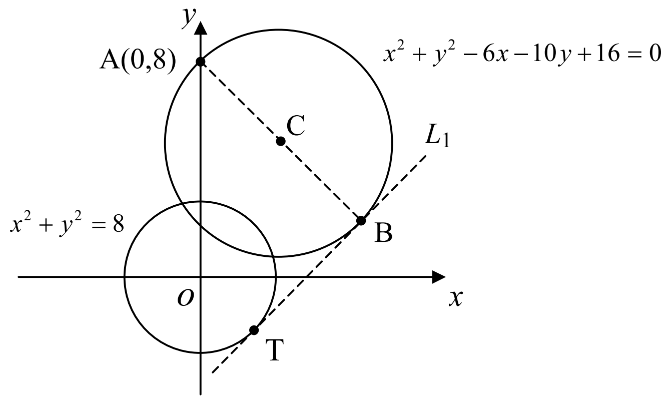
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. The diagram below shows two overlapping circles.
The larger of the two has as its equation $x^2 + y^2 - 6x - 10y + 16 = 0$ and the smaller $x^2 + y^2 = 8$.



(a) Write down the coordinates of C, the centre of the larger circle. **(1)**

(b) Hence find the coordinates of B, given that AB is a diameter of this circle. **(1)**

(c) The line L_1 is the tangent to the circle at B. Find the equation of L_1. **(3)**

(d) Show that the line L_1 is also a tangent to the smaller circle and establish the coordinates of T, the point of tangency. **(4)**

2. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + k$, where a and k are constants.

(a) Given that $U_0 = 6$, write down an expression for U_1 in terms of a and k. **(1)**

(b) Hence show that U_2 can be written as $U_2 = 6a^2 + ka + k$. **(1)**

(c) Given now that $U_2 = 0$, find k, where $k > 0$, if $6a^2 + ka + k = 0$ has equal roots. **(3)**

(d) Now find a when k takes this value. **(2)**

3. Solve the equation $2(2\cos 2x^\circ + \cos x^\circ) = -3$ in the interval $0 \leq x \leq 360$. **(5)**

4. In a marine tank the amount of salt in the water is crucial for the health of the fish. Recommended limits give a salt solution of between 41 and 55 grammes per gallon (g/gallon).

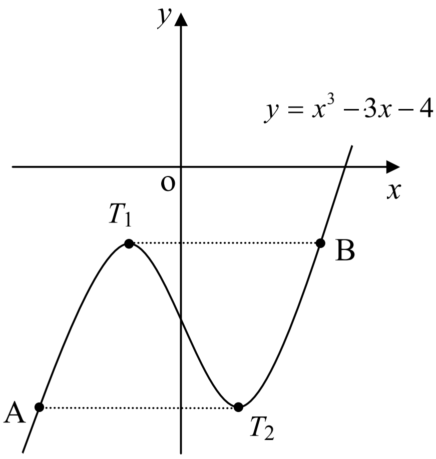
It is known that the strength of the salt solution decreases by 15% every day. To combat this, salt is added at the **end** of each day, which effectively increases the strength of the solution by 8 g/gallon, thus creating a closed system.

To allow the plants to acclimatise the initial strength in the tank has to be 45 g/gallon.

(a) For how many days should the system be run before the introduction of fish ? **(3)**

(b) In the long term will the strength of the solution remain within safe limits ? Give reasons. **(3)**

5. The diagram shows a sketch of the graph of $y = x^3 - 3x - 4$.



The tangents at the turning points of the curve meet the curve again at the points A and B as shown.

(a) Find the coordinates of the two stationary points T_1 and T_2. **(4)**

(b) Establish the coordinates of A and B. **(4)**

(c) Show that the tangents to the curve at A and B are parallel. **(2)**

6. The functions $f(x) = \dfrac{1}{\frac{1}{2}x + 1}$ and $g(x) = 2x^2 - 4$ are defined on suitable domains.

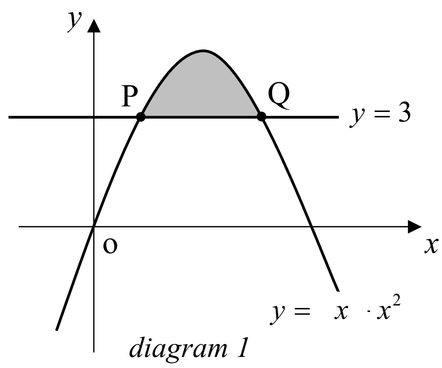
(a) Given that $h(x) = f\left(g(x)\right)$, show that $h(x)$ can be written as

$$h(x) = \frac{1}{(x-1)(x+1)}.$$ **(2)**

(b) State a suitable domain for $h(x)$. **(1)**

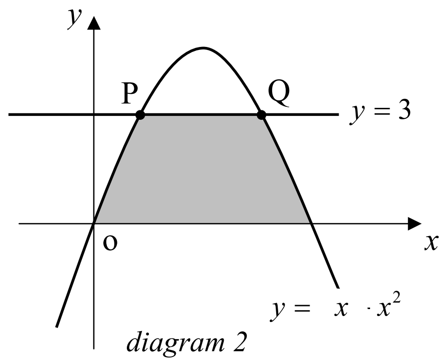
(c) Show that there are two values of x for which the functions f and h have the same image but that they are both irrational. **(4)**

7. The diagram shows a sketch of the curve $y = 4x - x^2$ and the line $y = 3$.



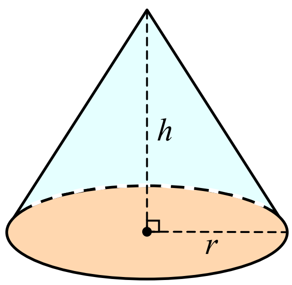
(a) Establish the coordinates of the points P and Q. **(2)**

(b) Calculate the shaded area in *diagram* 1. **(5)**



(c) Hence calculate the shaded area in *diagram* 2. **(3)**

8. A cone is such that the **sum** of its base **diameter** and its vertical height is 18cm.



(a) For this cone, write down an expression for the height (h) in terms of the radius (r). **(1)**

(b) Given that the formula for the volume of any cone is $V = \frac{1}{3}\pi r^2 h$, show that a function for the voulme of this cone can be expressed as

$$V(r) = 6\pi r^2 - \tfrac{2}{3}\pi r^3$$

(2)

(c) Hence find the value of r which will maximise the volume of the cone, and calculate this maximum volume in cubic centimetres. **(5)**

9. Two forces are represented by the vectors $F_1 = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $F_2 = \sqrt{3}\,\underset{\sim}{i} + \underset{\sim}{k}$.

Calculate the angle between these two forces. **(5)**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** C(3,5) **1 mark** •1 for extracting centre **(b) ans:** B(6,2) **1 mark** •1 establishing coords. of B **(c) ans:** $y = x - 4$ **3 marks** •1 for gradient of CB (or equiv.) •2 knowing $m_1 \times m_2 = -1$, and $m_{\tan}$ •3 for equation **(d) ans:** T(2,-2) **4 marks** •1 setting up a system •2 solving system correctly •3 stating 1 root (1 ans.) = a tangent •4 completing point T	(a) •1 C(3,5) (b) •1 B(6,2) (c) •1 $m = \frac{2-5}{6-3} = -1$ •2 $\therefore\ m_{\tan} = 1$ •3 $y - 2 = 1(x - 6)$ (d) •1 solve $\left.\begin{matrix} x^2 + y^2 = 8 \\ y = x - 4 \end{matrix}\right\}$ •2 $2(x-2)^2 = 0 \quad \therefore\ x = 2$ (twice) •3 written statement (1 ans., 1 point) •4 $y = 2 - 4 \ \therefore\ y = -2$, T(2,-2)
2.	**(a) ans:** $U_1 = 6a + k$ **1 mark** •1 substituting **(b) ans:** proof **1 mark** •1 correct substitution to ans. **(c) ans:** $k = 24$ **3 marks** •1 use of the discriminant •2 correct substitution (of *a*, *b* and *c*) •3 solving to answer **(d) ans:** $a = -2$ **2 marks** •1 setting up equ. to solve •2 solving to ans.	(a) •1 $U_1 = aU_0 + k \Rightarrow U_1 = 6a + k$ (b) •1 $U_2 = aU_1 + k$ $= a(6a + k) + k$ $= 6a^2 + ka + k$ (c) •1 for equal roots ..... $b^2 - 4ac = 0$ (stated or implied) •2 $k^2 - 4(6)(k) = 0$ •3 $k(k - 24) = 0 \ \therefore\ k = 24 \quad (k \neq 0)$ (d) •1 $6a^2 + 24a + 24 = 0$ •2 $6(a+2)(a+2) = 0 \quad \therefore\ a = -2$
3.	**ans:** $\{75\cdot5^\circ, 120^\circ, 240^\circ, 284\cdot5^\circ\}$ **5 marks** •1 correct double angle sub. •2 manipulation to factorising •3 first angle from first factor •4 first angle from second factor •5 remaining two angles	•1 $4(2\cos^2 x - 1) + 2\cos x + 3 = 0$ •2 $(4\cos x - 1)(2\cos x + 1) = 0$ •3 $x = 75\cdot5^\circ$ •4 $x = 120^\circ$ •5 $x = 240^\circ$, $284\cdot5^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**(a) ans:** 4 days **3 marks** •1 setting up recurrence •2 knowing to look at <u>low</u> value (before +8) •3 calculations and answer **(b) ans:** Yes (+ reasons from limits) **3 marks** •1 stating why limit exists •2 calculating limit •3 considering upper and lower limit in conclusion (own discretion)	(a) •1 $U_1 = 0.85(45) + 8$ •2 $U_1 = 0\cdot 85(45) = \underline{38\cdot 25} + 8 = 46\cdot 25$ •3 $U_4 = 0\cdot 85(48\cdot 21) = \underline{40\cdot 98} + 8 = 48\cdot 98$ next day low value will be > 41. (b) •1 limit exists because $-1 < a < 1$ •2 $L = \frac{b}{1-a} = 53\frac{1}{3}$ (or equiv.) •3 solution will always have a strength of between $45\frac{1}{3}$ and $53\frac{1}{3}$ g/gallon.
5.	**(a) ans:** $T_1(-1,-2)$, $T_2(1,-6)$ **4 marks** •1 knowing to differentiate •2 differentiating •3 solving for x coords. •4 completing points **(b) ans:** A(-2,-6) , B(2,-2) **4 marks** •1 attempting to solve for x •2 using synth. div. (or trial & error) for A •3 using synth. div. (or trial & error) for B •4 completing points **(c) ans:** $m_1 = m_2 = 9$ $\therefore$ parallel **2 marks** •1 for sub. x values into derivative •2 statement equal gradients are parallel	(a) •1 for S.P.'s $\frac{dy}{dx} = 0$ (stated or implied) •2 $\frac{dy}{dx} = 3x^2 - 3$ •3 $3(x^2 - 1) = 0$ $\therefore$ $x = \pm 1$ •4 $T_1(-1,-2)$, $T_2(1,-6)$ (b) •1 for A .... $x^3 - 3x - 4 = -6$, etc. •2 for A -2 | 1 0 -3 2 •3 for B 2 | 1 0 -3 -2 •4 A(-2,-6) , B(2,-2) (c) •1 @ A , $m = 3(-2^2) - 3 = 9$ @ B , $m = 3(2^2) - 3 = 9$ •2 since gradients are equal the two tangents are parallel
6.	**(a) ans:** proof **2 marks** •1 correct substitution •2 manipulation to answer **(b) ans:** $x \neq \pm 1$ **1 mark** •1 answer **(a) ans:** proof **4 marks** •1 equating functions •2 manipulation to quadratic •3 use of discriminant (or equiv.) •4 statement/conclusion	(a) •1 $f(g(x)) = \frac{1}{\frac{1}{2}(2x^2 - 4) + 1}$ •2 $... = \frac{1}{x^2 - 2 + 1} = \frac{1}{x^2 - 1} = \frac{1}{(x-1)(x+1)}$ (b) •1 $x \neq \pm 1$ (c) •1 $\frac{1}{x^2 - 1} = \frac{1}{\frac{1}{2}x - 1}$ •2 $\frac{1}{2}x - 1 = x^2 - 1 \Rightarrow x - 2 = 2x^2 - 2$ $\Rightarrow 2x^2 - x - 4 = 0$ •3 $b^2 - 4ac = 1 - (4(2)(-4)) = 33$ •4 roots are real, <u>distinct</u> and <u>irrational</u> (or equivalent explanation)

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** P(1,3) , Q(3,3) **2 marks** •1 for equating •2 solving and stating points **(b) ans:** $1\frac{1}{3}$ units^2 **5 marks** •1 setting up integral •2 for limits •3 integrating •4 for subst. numbers •5 calculating answer **(c) ans:** $9\frac{1}{3}$ units^2 **3 marks** •1 finding root for limit •2 calc. area between curve and x-axis •3 subtracting for answer	(a) •1 $4x - x^2 = 3$ (or equivalent) •2 $x^2 - 4x + 3 = 0 \Rightarrow x = 1 \;\; or \;\; x = 4$ $\Rightarrow$ P(1,3) , Q(3,3) (b) •1 $\int [(4x - x^2) - 3] \; dx$ *pupils may integrate between ordinates and subtract a rectangle* •2 $\int_1^3$ .... •3 $\left[2x^2 - \frac{x^3}{3} - 3x\right]_1^3$ •4 $(18 - 9 - 9) - (2 - \frac{1}{2} - 3)$ •5 $1\frac{1}{3}$ (c) •1 $4x - x^2 = x(4 - x) = 0 \;\therefore\; x = 4$ •2 $\int_0^4 4x - x^2 = 10\frac{2}{3}$ •3 $10\frac{2}{3} - 1\frac{1}{3} = 9\frac{1}{3}$
8.	**(a) ans:** $h = 18 - 2r$ **1 mark** •1 answer **(b) ans:** proof **2 marks** •1 knowing to substitute for h •2 processing to answer **(c) ans:** $r = 6cm$, $V = 72\pi \;\; or \;\; 226\cdot 1$ cm^3 **5 marks** •1 method (differentiation) •2 differentiation •3 solving for r •4 proving a maximum (nature table) •5 calculating V (multiple of π or not)	(a) •1 $d + h = 18 \Rightarrow 2r + h = 18$ $\therefore\; h = 18 - 2r$ (b) •1 $V = \frac{1}{3}\pi r^2(18 - 2r)$ •2 $V = 6\pi r^2 - \frac{2}{3}\pi r^3$ (c) •1 $V'(r) = 0$ *at* max. (stated or implied) •2 $V'(r) = 12\pi r - 2\pi r^2$ •3 $2\pi r(6 - r) = 0 \;\;\therefore\;\; r = 6$, ~~$r = 0$~~ •4 nature table showing a maximum •5 $V(6) = 216\pi - 144\pi = 72\pi$ cm^3
9.	**ans:** $75\cdot 9^\circ$ **5 marks** •1 for dealing with unit vector notation •2 magnitude of F_1 •3 magnitude of F_2 •4 for scalar product •5 for answer	•1 $F_1 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $F_2 = \begin{pmatrix} \sqrt{3} \\ 0 \\ 1 \end{pmatrix}$ •2 $F_1 = \sqrt{4 + 1 + 4} = 3$ •3 $F_2 = \sqrt{3 + 1} = 2$ •4 $F_1 \,.\, F_2 = 2\sqrt{3} + 0 - 2$ •5 $75\cdot 9^\circ$

Total 67 marks